TEACHING DOSSIER

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Table of Contents

1. Brief Biography

My teaching experience begins in 2011 as a Master’s student in Mathematics at the Universidad de los Andes where I was a teaching assistant for several first and second years courses. After I earned my Master’s degree in 2013, I had the opportunity to teach several introductory courses at the Universidad de Los Andes as well as other two universities in Bogotá, Colombia. These courses included Calculus, Algebra, Logic and Probability. In 2015 I began my doctorate degree in Mathematics at the University of Western Ontario where I have been a teaching assistant for more than 10 different courses. I have also been actively involved in the development of the Directed Reading Program at the Department of Mathematics mentoring undergraduate students into interesting mathematical projects and help them to achieve their goals pursuing a major in this area. Finally, my background and diverse experience have help me to success as a passionate teacher; as it is outlined in this Dossier, I enjoy teaching and look forward to developing my skills further as I pursue my academic career.

2. Teaching Philosophy

Since I was in high school, I was passionate about mathematics, yet not only about learning and applying new tools everyday; in fact, passionate about helping my classmates to go throughout their difficulties to succeed in class. I remember my professor back then saying: “you are able to understand a topic whenever you are able to explain it to others and help them learn”, and this philosophy has driven my career up to the current point where I am now: Finding a balance between researching and teaching, and these two pillars interconnect with each other to create a harmony in my career. All of this boosts my enthusiasm doing mathematics, and this not only means researching mathematics, it also includes giving enthusiastic lectures, preparing and designing interesting assignments and projects for students and last but not least, interacting with students either in class or office hours.

Throughout my career, I have had the opportunity to work with a diverse class of students. I have been fortunate to interact with students within a huge range of ages, socioeconomic and academic backgrounds, and full-time/part-time status who have opened my eyes to the important role of the instructor in the learning process of each person. Even though any student's learning requires specific approaches, understanding the common problems and biases towards mathematics allows me to model my teaching style in the most beneficial way for each audience and provide meaningful lectures and assessments for the students to succeed, not only during the course, but also to obtain an useful insight of mathematics for their particular interests. Therefore, I have realized that many students share a similar mindset about mathematics in general: ***Mathematics is a hard subject in nature***, and it is only meant to geniuses or brilliant students. ***Mathematics is useless*** unless one wants to become a scientist and most of the things covered in the courses will not be used in “real life”. ***Mathematics is uninteresting and boring,*** the more one study, the more abstract and deep mathematics turns itself, and it is usually hard to relate the concepts to each others. Therefore, *my teaching philosophy relies on fighting against these stereotypes*; being aware of the world's need for people who estimate, verify, support, interpret results and connect with other's ideas are the pillars that drive my career and have lead me to the teacher that I currently am.

Firstly, I believe that ***Mathematics is a feasible subject*** that everyone is capable of achieving any level of understanding with the right guidance. I admit in front of my students that it might not be an easy subject, I share stories of my struggles as student and I make them feel that making mistakes is not wrong. I try to select problems carefully that leads student to fully analyze and appraise theorems and results discussed in lectures; always considering an accessible level for everyone in the course. Since I was a teaching assistant, I have gathered the most common mistakes made by students in the tests or assignments, I discussed with them where is the root of the problem, because there are even some voids from high-school or previous courses that need to be filled. I also tend to do a survey at the beginning of class to gather people's previous knowledge and expectations for the particular course; this survey can be presented also with a small problem set to keep track of those topics that need to be addressed beforehand. Once this information is collected, I suggested reading lectures and problems for people to keep up, and I devoted an extra teaching sessions or office hours to help as most students as I can for them to catch up. I support the idea that learning mathematics depends on student's opportunities, experiences and effort, and not in a innate intelligence. I am a fierce supporter of the premise that all students are capable of participating and achieving in mathematics, and every single one deserves support to reach to the highest levels.

Secondly, I work for presenting ***Mathematics as a useful subject***. Even though many students will not compute an integral or apply the fundamental theorem of algebra on an instance of their future; I point out how mathematics enhances the ability to think rationally and to clearly organize ideas which leads to practical applications. I share my passion about Mathematics in the classroom, I show enthusiastically the endless implications and purposes of the particular theory that I have been teaching. I also refer to mathematical facts as artwork; and that in fact, mathematics is another form of art to appraise. I like to raise the analogy of this subject with other forms of art to my students: not all people like art paintings, sculptures or literature, but they do agree that this is some form of natural human expression; and Mathematics is not far from that premise; it is the natural language that the universe uses to manifest among us.

Finally, ***I make of mathematics a lively and exciting subject***. I base my teaching methods, style and tactics on encouraging students to adopt a growth mindset, to see mathematics as a subject of beauty and creativity in which anyone can thrive, where speed is not relevant, but depth is. I point out that the journey to an answer can be as important as the destination, and I adopt an approach where sense-making matters more than memorization. I make use of technological tools to help learner's experience, I discuss examples in class, showing problem-solving techniques, and making mistakes, getting stuck, and trying different strategies as normal steps in the process of learning. These combinations of the theoretical with the practical mindset, the traditional lectures with active-learning classroom strategies bring mathematics to light as a gratifying and pleasant subject among students.

As a final point, I completely perceive teaching as a process that goes beyond the classroom; it is a whole compendium of one's preparation, experience, motivation and philosophy that reflects on student's mindset towards the subject, and each day I pursue an improvement on every aspect. I regard teaching as an essential stage of learning and forming character; and more than teaching the contents of a syllabus during a course, I establish on my teaching philosophy and methods the significance of the teaching-learning connection based on strengthening the process of reading, interpreting, applying and explaining on each student. Even though some of the students will eventually forget the explicit contents of the course, I want to be a professor that prints a permanent mark on each student's professional and personal development, and one that they will always remind.

3. Teaching Experience

3.1. University of Western Ontario: Teaching Assistant (2015-2020)

- *Mathematics 1600: Linear Algebra I (Fall 2019, Winter 2019)*

First year linear algebra course offered for students intended to major in the Faculty of Science. The main contents of the course are: properties and applications of vectors; matrix algebra; solving systems of linear equations; determinants; vector spaces; orthogonality; eigenvalues and eigenvectors.

The course consisted of an approximate 200 students and the main lectures held twice per week during 12 weeks. The students are divided into 6 or 7 tutorial groups of approximate 30 students each. Each group meets one hour per week and I was in charge of running two of these groups. My main roles during these sessions were to review material from the course, answer questions, and run group activities such as problem-solving suggested questions for exam preparation. There were weekly meetings with the instructor and other teaching assistant to discuss the progress of the groups and discuss potential problems to bring to the session. An example of a worksheet of prepared problems to the sessions can be found in Appendix A.

- *Mathematics 3124: Complex Analysis (Fall 2019)*

Third year course offered to students pursing a major in pure Mathematics. The main contents of this course are: The Cauchy-Riemann equations, Cauchy’s integral theorem and formula, the identity theorem, the maximum modulus theorem, Taylor and Laurent expansions, isolated singularities, the residue theorem and the argument principle.

The course consisted of an approximate 25 students and the main lectures held twice per week during 12 weeks. My main role was to provide effective feedback to the weekly suggested problem set sent by the instructor. The grade for these assignments only serves as feedback; they did not affect the final grade. I was also in charge of grading the exams.

- *Math 3159B: Introduction to Cryptography (Winter 2019)*

Third year course offered to students pursuing a major in Mathematical sciences. The main contents of this course are: Elementary Number Theory, Introduction to Computational Complexity, Discrete Logarithm Problem and Diffie-Hellman Key Exchange, RSA Encryption, Primality Test, Factorization Algorithms, and Quadratic Reciprocity, Elliptic Curve Cryptography.

The course consisted of an approximate 30 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly quizzes as well as the exams. I also held office hours once per week intended to help students with questions from the lectures.

- *Mathematics 1120: Fundamental Concepts in Mathematics (Winter 2019)*

It is an introduction to rigorous mathematical thinking. The main purpose of the course is to teach students to understand mathematical reasoning and write mathematical proofs. Primarily intended for students interested in pursuing a degree in one of the mathematical sciences. The contents of this course include logic, set theory, relations, functions and operations, careful study of the integers, discussion of the real and complex numbers, polynomials, and infinite sets.

The course consisted of an approximate 50 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets.

- *Mathematics 3020: Introduction to Abstract Algebra (Fall 2018)*

Third year course offered to students pursuing a major in pure Mathematics. The main contents of this course are: numbers and polynomials, binary operations and useful axiom of rings and ideals, factorization, introduction to field theory, introduction to groups.
The course consisted of an approximate 30 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets.

- *Math 1229: Methods of Matrix Algebra (Winter 2018, Winter 2016)*

First year course offered to any student in the university and affiliate Colleges. The main contents of this course are Vectors in ℝm; Equations of lines and planes; Linear Equations; Solution of Linear Systems; Matrix Algebra; Matrix Multiplication and Inverses; Determinants.
The course consisted of an approximate 300 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set and assignments sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets.

- *Calculus 2302: Intermediate Calculus I (Fall 2018, Fall 2017)*

Second year course offered to students pursuing a major in any of the Mathematical sciences. The main contents of this course are: three-dimensional analytic geometry, quadric surfaces, vector functions and space curves; arc length, curvature and Differential calculus of functions of several variables.
The course consisted of an approximate 40 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set and assignments sent by the instructor as well as the exams. I also held

office hours once per week intended to help students with questions from the lectures or problem sets.

- *Calculus 1501*: *Calculus II for Mathematical and Physical Sciences (Winter 2017)*

First year course offered to students pursuing a major in any of the Mathematical sciences. The main contents of this course are: techniques of integration; The Mean Value Theorem and its consequences; series, Taylor series with applications; parametric and polar curves with applications; first order linear and separable differential equations with applications.
The course consisted of an approximate 70 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set and assignments sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets.

- *Calculus 0110*: *Introductory Calculus (Winter 2016)*

First year course offered to any student in the university and affiliate colleges that need to strength their mathematical bases from high school. The main contents of this course are: Limits, continuity, definition of derivative, rules for differentiation, higher-order derivatives, velocity, acceleration, implicit differentiation, related rates, exponential functions, logarithmic functions, differentiation of exponential and logarithmic functions, maxima and minima, concavity, curve sketching, optimization.
The course consisted of an approximate 100 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set and assignments sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets

- *Calculus 1000: Calculus I (Fall 2015, Fall 2016)*

First year course offered to any student in the university and affiliate colleges. The main contents of this course are: Review of limits and derivatives of exponential, logarithmic and rational functions. Trigonometric functions and their inverses. The derivatives of the trig functions and their inverses. L'Hospital's rules. The definite integral. Fundamental Theorem of Calculus. Simple substitution. Applications of integration, including areas of regions and volumes of solids of revolution.
The course consisted of an approximate 200 students and the main lectures held twice per week during 12 weeks. My main role was to grade and provide effective feedback to the weekly suggested problem set and assignments sent by the instructor as well as the exams. I also held office hours once per week intended to help students with questions from the lectures or problem sets.

- *Mathematics General Help Centre (4 hours / week)*

The Math Help Centre is a space offered by the Department of Mathematics for all first- and second-year students to get help with any course in mathematics. The help centre is available 5 days a week during the afternoon (1pm-5pm) and it staffed by graduate students with a passion for teaching. Students come to the help centre to deal with specific questions, review from lectures, or they use it as a study space with help on hand in case they get stuck on.

3.2. Universidad de los Andes: Instructor and Teaching Assistant (2011-2015)

I served as an instructor for the following courses at Universidad de los Andes in several opportunities. Uniandes is a private university in Colombia ranked among the top-10 in Latin America. The student population consists of the Top 1% high school graduates and they enjoy a diversity of backgrounds in terms of race, socioeconomic status, religion, sex. The university provide strong economic support to the top students with financial needs.

In terms of first year courses taught at Uniandes, each course enrols more than 200 students which are divided into small groups of approximate 35 students which are assigned to a particular instructor. The program outline, schedule and exams are in charge of the course coordinator and meetings among all the instructors are held two weeks before each exam to discuss and review the topics covered as well as suggest potential problems to include in the test papers. The teaching load of these courses usually involved 1.5-hour class twice per week during 16 weeks for a group of 35 students.

I was responsible for all the preparation of the lecture material, assignments, quizzes and assessment of the following courses.

- *Integral Calculus and Differential Equations*

  It is a second semester calculus course for students who have previously been introduced to the basic ideas of differential and integral calculus. The main topics of this course are: applications and techniques of integration, infinite series and the representation of functions power series, and differential equations. Three midterms exam are held during the semester (20% each) an accumulative final exam (25 % each) and the remaining 15% was open to the criterion of each instructor. In my case, I delivered assignments (5% total) and apply quizzes (5% total) before each exam. The remaining grade 5% was devoted to attending class and participating, which is an advantage to use in a class of small groups.

- *Multivariate Calculus*

  It is a second-year calculus course for students who have previously been introduced to differential and integral calculus in one variable, parametric equations and notions of differential equations. The main topics of this course are: three-dimensional geometry, partial derivatives, gradient divergence and curl; line and surface integrals; and the theorems of Green, Stokes, and Gauss. Two midterms exam are held during the semester (30% each) an accumulative final exam (25 % each) and the remaining 15% was open to the criterion of each instructor. In my case, I delivered assignments (5% total) and apply quizzes (5% total) before each exam. The remaining grade 5% was devoted to attending class and participating, which is an advantage to use in a class of small groups.

- *Mathematics for Biology and Medicine*

  It is a second year course for students pursing a major in biology, chemistry, health sciences and related careers. The main objective is to learn and apply mathematical concepts particularly into biology and medicine. The approach to the mathematical theory is done through problem discussion. The main topics in this course are: matrix algebra, introduction to statistics (measures of central tendency) and probability (random variables, binomial and normal distributions). The used reference was Neuhauser, Claudia: Calculus for Biology and Medicine, Prentice Hall. Two midterms exam are held during the semester (30% each) an accumulative final exam (25 % each) and the remaining 15% was open to the criterion of each instructor. In my case, I delivered assignments (5% total) and apply quizzes (5% total) before each exam. The remaining grade 5% was devoted to attending class and participating, which is an advantage to use in a class of small groups.

I served as a teaching assistant in several first-year courses (pre-calculus, differential calculus, linear algebra) during my Master's degree providing support to the main instructor by grading quizzes, assignments, exams and running tutorial sessions. I also had a major assistant role in the following course.

- *Numerical Analysis and Complex Variables.*

It is third year course intended for students pursing a major in engineering. The main topics covered in this course are: Fourier Analysis, Partial Differential Equations, Laurent Series, Numerical Methods for integration and solving differential equations. Students also use MATLAB for solving the suggested problems and developing a final project. My main role was to run MATLAB tutorial sessions, review lecture material, grade assignments, exams and the final projects.

Instructors and Teaching Assistant provide help in any course in mathematics to any student that attends the *Pentagon* (Mathematics Help Centre) which runs Monday to Friday from 8am to 4pm. Every instructor/teaching assistant stays in average 4 hours/week in this help space.

3.3. Universidad del Rosario: Instructor
(2013)

During 2013 I served as an instructor at la Universidad del Rosario in the growing department of mathematics at that institution. Universidad del Rosario is a private university in Colombia known for its strong programs in business, administration and economics; therefore, the direction of the courses was driven towards that area. I was the main instructor for 2 courses each semester, helping to develop the program outline and assessment methods. Each course consisted of approximate 150 students divided into sections of around 35 students each. The lectures were held in two 2-hour sessions per week during 16 weeks. Even though the course assessment was the same for all sections, each instructor had the freedom to develop their own assignments, quizzes and midterm exams; as well as responsible for all the preparation, grading and evaluation. The final exam was made considering an end-of-the-term meeting among all the instructors for the course.

- *Probability*

I was the instructor for two groups of 30 students each semester for this course. The main contents of this course are: introduction to statistics (measures of central tendency), combinatorics, random variables, discrete and continuous distributions, sampling and hypothesis testing. Three midterms exam are held during the semester (20% each), an accumulative final exam (25 % each) and the remaining 15% was open to the criterion of each instructor. In my case, I delivered assignments (5% total) and apply quizzes (5% total) before each exam. The remaining grade 5% was devoted to attending class and participating.

- *Matrix Algebra and Linear Optimization*

I was the instructor for one group of 25 students each semester for this course. The main contents of this course are: Matrix operations, Gauss-Jordan elimination, Inverse Matrices and Determinants, Linear programing, the simplex method and sensitivity analysis. Applications to real world problems Two midterms exams are held during the semester (20 % each), assignments (15 % total), class participation in case-study problem (15% total) and a final project (30% total). The final project was an independent activity for groups of 4-5 students applying the main contents of the course into optimization problems in the Colombian society. A list of suggested problems is prepared by the instructor and students choose what is more appealing for them. A final report (15%) and group presentation (15%) was held at the end of the semester. See *Appendix A* for a sample (in Spanish) of such reports.

3.4. Universidad Central de Colombia: Instructor **(2013)**

During 2013 I also served as an instructor at la Universidad Central de Colombia in the department of engineering and mathematics at that institution. I belonged to the roster of Instructors for the night-time and weekend programs; these programs are offered to mature and full-time working people that pursue an undergraduate degree in engineering. Classes for these special programs are held Monday to Friday from 6pm – 10pm and Saturdays from 9am – 5pm during 16 weeks. I was the instructor for two courses each term in a class sizes of approximate 30 students each. The methods of evaluation followed for this particular program were the following: Two midterms exam are held during the semester (30% each) an accumulative final exam (40 % each). Assignments are handed out every two weeks with carefully selected problems for the students to practice; and 80% of the exam questions are slightly different from the proposed problem sets.

The courses that I taught at this institution are the following.

- *Multivariate Calculus*
- *Mathematical Logic*
- *Linear Algebra.*

I also was a secondary instructor providing independent tutorial sessions for reviewing lectures and problem-solving to full-time students in the morning and afternoon programs in the following courses.

- *Integral Calculus*
- *Differential Calculus*

The activities and participation on these tutorial sessions were 15% of the final grade.

4. Teaching Effectiveness

4.1. Formal Teaching Evaluations

Most of the teaching evaluations at the University of Western Ontario are in charge of the instructor of the course. See the Appendix for a sample of such reviews. The aspects of assessment and the respective average obtained are the following.

Category	Average (out of 5)
Assessment on Teaching Assistant's Marking performance	4.7
Assessment on Teaching Assistant's Teaching Performance	4.5
Assessment on Teaching Assistant's effectiveness in completing assigned tasks.	4.9
Assessment on Teaching Assistant's promptness on arriving to the classroom, exams, office hours, etc.	5

At the Universidad de los Andes, Instructor performance is evaluated under the following categories scored discretely from 1 to 4, representing strongly disagree, disagree, agree and strongly agree respectively. For a class of 27 students in the course of *Integral Calculus and Differential Equations* the obtained results are as follows (Originally in Spanish).

Category	Course Average (out of 4)
The course meets the established learning objectives.	3.58
The development of the course foster student's commitment to studying this subject.	3.39
The instructor implement effective activities (presentations, discussions, projects, assignment, etc) to enhance student's learning.	3.52
The textbook, lecture materials or any other teaching tools used both inside and outside of the classroom are adequate for the course objectives.	3.48
The assessment methods are clearly established from the beginning of the course.	3.68
The exams, assignments and graded activities are appropriate and consistent with the course objectives.	3.61

The grades are returned accordingly to the university policies	3.53
The instructor is respectful with the students.	3.88
The instructor arrives promptly to the course lectures.	3.75
The instructor promptly assist students in both inside of the classroom and outside of it (office hours, email, e-learning platform, etc)	3.78

4.2. Informal Teaching Evaluations

“Sergio’s lectures begin with easy and relatable examples, he uses visual aids and constantly reminds the learning outcomes for the session”

¨The way that Prof. Chaves addresses topics is a logical flow from simple to complex concepts”.

“He does very good explanation of difficult problems by using simple examples”

“Prof. Chaves lectures are fun and well organized, he does a lot of group activities during the entire lesson”

4.3. Peer Evaluations from Class Observations

“Excellent usage of Audio-Visual aids to facilitate learning. Superior Enthusiasm displayed in delivering the lecture content. Effective usage of whiteboard around the classroom. Higher comfort level experienced between the attendees and the instructor leading to better understanding of the subject content, distribution of worksheets that helped the attendees understand the subject better. Overall, you practices an outstanding university teaching style and incorporates many teaching strategies that makes learning experience worthwhile.”
Glen DSouza. Research Assistant. Department of Chemical and Biochemical Engineering. (2019).

“You had an astute ability to explain complex concepts to your audience by deconstructing big ideas into smaller ones. When introducing the topic, you began by asking your students very simple questions, such as “what is a polygon?” and subsequently asking students to identify the correct polygons out of a variety of mixed shapes. This provided students with not only a clear definition but an opportunity to practice applying their knowledge. You continuously utilized interactive exercises with the class with handouts and gave the class adequate time to practice on their own. By taking up the worksheets after with the class, students were able to identify their own knowledge gaps and develop a clearer understanding of the content.”
Hanna Roberts. MSc. Candidate, Clinical Anatomy. Dept. of Anatomy and Cell Biology (2019).
See the Appendix for the complete evaluation letter.

5. Curriculum Development

5.1. Course Design and Teaching Innovations

I participated in the implementation of the contents of the course *Matrix Algebra and Linear Optimization* at Universidad del Rosario. I also have a proposed syllabus for a upper-year mathematics course that I developed as a PhD student at the University of Western Ontario. See the Appendix for a sample of these syllabus.

During most of my time as Instructor and teaching assistant, I prepared independently problem sheets, assignments and quizzes considering the topics covered in class and the most beneficial questions for the students to prepare for the exams.

5.2. Mentoring Students

I participated in the *Directed Reading Program* at the University of Western Ontario. The Directed Reading Program (DRP) pairs undergraduate students with graduate students/junior faculty to undertake independent study projects. It is intended to help motivated students explore topics in more depth than possible in a classroom setting.

Each project is for the duration of one academic semester, which is roughly thirteen weeks. Undergraduates can apply for DRP positions in the beginning of each term and those who are selected will be paired with mentors according to their mathematical interests and availability. The projects are based around the self-paced reading of a particular book or article with substantial guidance from the mentor, with the specific topic arrived upon by discussion of common interests between the mentor and the mentee.

My mentee, an engineering students passionate about mathematics but then-unsure for pursing a math major, and I worked on a geometrical approach to the Poincaré-Hopf theorem for surfaces in the 3-dimensional space. The followed reference was Firby, Peter A., and Cyril F. Gardiner. *Surface topology*. Elsevier, 2001. By the end of the term, my mentee was able to understand and explain the Euler Characteristic and the proof of the index theorem using a geometrical approach; moreover, he decided to completely dedicate to pursue his Major in mathematics after the discussion, advices and suggestions that I gave to him during our meetings. We will be working next term on a reading course in *Topology* following the classical book of Munkres.

5.3. Reflection on Teaching Mathematics

Teaching mathematical proofs for undergraduate students is a challenge for both instructors and students. Proof is an essential part on learning as it develops reasoning skills, improves problem solving aptitudes and it is also a fundamental basis for future researchers. This reflection is focused on studying the difficulties experienced in the learning and teaching process of formal mathematical proofs, and discuss whether the traditional "definition-theorem-example" lecture method is the most appropriate for the students to success in mathematical courses.

See the Appendix for the complete reflection on this thinking done as requirement for my professional development degrees.

5.4. Other Activities

During 2015 I volunteered as instructor of conversational basic Spanish for students travelling to Central and South America Spanish-speaking countries under the program Impact Experience program at Western. Students are part of projects that impacts a community abroad. Impact Experience offers all students at Western and its affiliates access to unique co-curricular opportunities to support community projects around the world.

During 2014 I worked as a reviewer for the bank of mathematical questions at the Colombian Institute for the Assessment of Education *ICFES*. This institution is the Colombian public entity in charge of developing, implementing, facilitating and applying the standardized exams to students prior to graduating from elementary, middle and highschool across the Country. The *ICFES* test is nationally recognized as the most important test in Colombia since it qualifies students according to their actual academic skills and therefore it can affect the possibilities that a student might have to be accepted in College.

6. Professional Development

During 2019 I pursued the Western Certificate in University Teaching to enhance my teaching career. It is intended to improve teaching skills for graduate students and postdoctoral scholars at the University of Western Ontario, and to prepare them for a future faculty or professional career.

6.1. Teaching Workshops and Conferences

Constructing Your Teaching Dossier Webinar

Webinar to learn about what goes into a Teaching Dossier. We will discuss the purpose/use of dossiers in academia and plan how to get started gathering information for your own document.

Strategies to Help Students Learn How to Learn

Session to learn about self-regulatory learning and meta cognition. You will learn different strategies to help your students be independent learners to optimize their learning process.

Increasing the Power of PowerPoint

This session aims to identify common challenges to student learning related to PowerPoint design and articulate key design principles inspired by research on multimedia learning theories. Participants are encouraged to bring a section of a PowerPoint presentation (either hardcopy or on a device) to the workshop. You will have the chance to discuss and apply design principles to story boarding and revising the slides in order to maximize their (and your) impact.

How to Care For and Use Your Teaching Voice

Being a TA brings a variety of vocal challenges. Class size, your confidence with the material, the interest or engagement of the students, the nature of the material itself, workload, and the use of amplification are just some of the variables that place different demands on the same vocal instrument. At times, it may feel like there are overwhelming barriers to finding a confident voice that fills space. This workshop is designed to fill that gap. By participating in this workshop you will learn: basic anatomy, how we create sound, and voice care strategies. Through an experiential portion, you will be introduced to producing an easy, confident teaching voice that is resistant to fatigue.

Assessing and Articulating your Teaching Skills for Future Careers

Reflect and assess the skills gained and outcomes achieved through GTA experience. Articulate skills developed to prospective employers

How Can Teaching Help Me Get a Job?

This panel discussion is intended to learn how former Western graduate students have relied on their teaching skills to succeed in their jobs after graduate school. Doctoral students will have a chance to reflect on their own skills and consider how to meaningfully communicate those skills. This workshop will also help participants to identify the areas in which they would like to grow as instructors.

Case Studies on Teaching

This interactive session is meant to practice working through challenging teaching scenarios. The cases studies presented in this session were developed by Western Graduate students and are based on authentic TA experiences. You will have the chance to apply creative problem-solving skills to several cases in small groups of interdisciplinary peers.

Writing Diversity Statements (Own Your Future)

Diversity statements are living documents that allow instructors to reflect on their contributions to diversity and inclusion within the context of their teaching. In this session, we will discuss typical components of diversity statements (an increasingly common component of academic job applications) and how you can address these components in a diversity statement of your own or as part of your teaching dossier.

Writing Teaching Philosophy Statements – WEBINAR

Participate in this online session to learn about the key components of a teaching philosophy statement and start drafting your own. Registrants will be added to an OWL site where they will be able to join the webinar using Blackboard Collaborate.

Fall Perspectives on Teaching Workshop

Offered twice a year, Perspectives on Teaching is a full-day conference designed to showcase teaching innovations at Western, and introduce instructors to best practices in student-centered instruction which can enhance the student experience. The conference program typically includes a keynote address and 6-9 concurrent sessions.

Tried and True Strategies to Address Common Assessment Conundrums

Love teaching but hate marking? Drowning in student papers, exams, and lab reports? Tired of offering the same kinds of assessment tasks to your students? Looking for some new approaches to gauge what students know and can do? During this interactive session, Dr. Crocker will connect the current literature on assessment for, of, and as learning while sharing approaches to consider when confronted by common assessment conundrums.

Effective Practices for Peer Review of Teaching: Moving Beyond the Student Perspective

Student feedback is a valuable resource for reflecting on your teaching but it draws on only one perspective. Faculty colleagues can also provide unique insight into your approaches to teaching and course design practices. Join this interactive session to discuss the who, what, when, where, why, and how of getting peer feedback on your teaching. We will start by exploring models of peer review and their benefits. We will also discuss effective processes for collecting feedback in the classroom, online environments, and on course structure/materials. In small groups, participants will generate guiding questions for obtaining feedback and assess existing templates for peer review. Participants will leave the session with a set of resources that will guide both the collaborative process of getting feedback and ways of implementing self-reflection strategies.

Flexible Assessment: Designing Effective Assessments that Support Student Wellness

Western is dedicated to a thriving campus, which involves promoting the mental health and wellness of our students. This commitment is increasingly important as more and more students are reporting that anxiety and stress are having a negative impact on their academic success and there is a corresponding upsurge in demand for wellness support services. The way we design our courses, including assessment methods, processes, and policies, can have a substantive impact on the wellness of our students. In this session, we will discuss practical ways instructors can maintain or improve the design and administration of assessments that promote wellness and deep learning while maintaining the integrity and rigor of our pedagogical practices.

New Policies for Student Absence and Accommodation -- Is Your Syllabus Ready?

Western has two new policies that guide academic accommodation for students. Join us to ask questions, explore how the new policies will affect how you plan for makeup exams and alternative assignments, and hear what strategies faculty colleagues are planning to implement as they prepare to support students in large and small classes in different disciplines.

6.2. Courses and Other Development Activities

TA Day: Graduate Student Conference on Teaching

This one-day yearly conference introduces graduate students to teaching at Western and helps prepare them for their roles as Teaching Assistants. TA Day has welcomed new graduate students to Western for over 30 years.

Conference highlights include a keynote presentation by an award-winning faculty member and a panel of experienced Teaching Assistants from across the disciplines. Participants choose from a variety concurrent workshops that focus on different teaching topics including: facilitating discussions, managing difficult situations, communication strategies, and more. Graduate students also have the opportunity to hear from key campus partners including Western Libraries, the Writing Support Centre, Learning

Skills Services, the Wellness Education Centre, the TA Union (PSAC Local 610), and the Society of Graduate Students.

Advanced Teaching Program

The Advanced Teaching Program (ATP) is a 20-hour short course designed for advanced graduate students who would like to develop practical teaching skills for current and future teaching roles. Topics include course design strategies, active learning, authentic assessment of student learning, and maintaining a culture of respect and community in the classroom. Participants gain hands-on experience by practicing instructional techniques in microteaching sessions where they receive constructive feedback from peers and an experienced team of instructors.

Teaching Mentor Program

The Teaching Mentor Program is a unique opportunity for graduate students and postdoctoral scholars to receive feedback on their teaching and classroom management approaches from peers in their own teaching environment. Four to five participants will work together and visit each other's classes, tutorials or labs over the course of the semester. I was part of a group of 5 Graduate Student from a diverse range of backgrounds (Engineering, Medical Sciences, Humanities and Science).

Refer to the Appendix for a sample of the feedback received as well as one of the feedback provided as observer.

Appendix: Additional Files

Teaching Assistant performance review

Department of Mathematics: TA evaluation form

The purpose of this TA Evaluation Form is to keep track of how our TAs are doing and to give them feedback to help them improve. This information can be used when evaluating TAs for awards, when writing letters of reference regarding teaching, when assigning TA duties in future terms, and when considering disciplinary action. It is therefore useful to have both positive and negative comments recorded. Note that the TAs will be provided with copies of these forms.

Semester: Fall 2018 TA name: Sergio Chaves

Course: 3020A Instructor: David Riley

Summary of the TA's duties: Marking, TeXing assignment solutions, weekly office hours.

Answer the following questions on a scale of 1 to 5, with 5 the best score. You may also write N/A.

Were you happy with the TA's marking? 5

Were you happy with the TA's teaching/help centre? 5

Was the TA prompt in completing assigned tasks? 5

Did the TA arrive promptly to exams, tutorials, help centres, etc? 5

Please provide any other comments, positive and/or negative, continuing on the reverse side if necessary:

Instructor signature: Date: 24 January 2019

MATH 4015
Transformation groups
Fall 2020
Course Syllabus

Course Description

This course is intended as an application of the *theory of groups* from the abstract algebra course and the course of *general topology* combining main results from both theories to study invariant properties of spaces through its symmetries or actions of groups. This course is available to any student who have the prerequisites; however, it is focused for students majoring in Mathematics.

Course Schedule and Location

The lectures will meet on Wednesday - Friday : 2:30 - 4:00 pm in the room MC 108.

Prerequisites

Either one of *Math 3020: Introduction to Abstract Algebra* or *Math 3120: Group Theory*, and *Math 3121: Topology*.

Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

Learning Outcomes

Upon completion of this course,students will be able to:

- Use the main theorems from Group Theory and general Topology to characterize transformation groups on topological spaces.
- Categorize and distinguish the fibration constructions in transformation groups: Classifying spaces, Principal and Fiber Bundles.
- Identify the Borel construction as an invariant of spaces and functions that preserve the group action.
- Apply fundamental theorems of the theory of G-space to describe invariant geometrical properties of spheres and projective spaces.

- Evaluate the rigidity of transformation groups by identifying if the action is in a topological environment or in a differential environment.

Methods of Evaluation

The following methods will be used as the assesment tools for grading this course:

- **Assignments:** They will be a worksheet of 5-8 problems that will be addressed by applying the theories discussed in the lectures. Assignments will be assessed according to the accuracy, completeness, and thoughtfulness of your responses to the assignment questions. Each assignment is distinctly different and is an individual student effort. It is expected that the solutions are typed in LaTeX format. The due date of each assignment can be found in the course timetable. *No late assignments will be accepted*
- **Participation:** Student participation can substantially enrich the learning experience for both the students and the instructor. Discussion will be encouraged throughout the course, yet effective participation requires you to read assigned readings before coming to class. You are expected to attend all classes. Please notify me in advance if you will need to miss a class.
- **Exams:** The midterm exam is a 1.5 hr paper based test, the scheduled time for the midterm exam can be found in the course timetable. The final exam is a 2 hr paper based test, it will be scheduled by the Registar Office to take place during the december exam period, *Do not arrange traveling during these dates.* Midterms are non-cumulative, but the final exam is based on the entire syllabus for the course. If a student misses the midterm exam, that student shall provide a documented excuse or a mark of zero will be entered for that midterm. *There will be no make-up midterms*, and the weight of the missed midterm will be transferred to the final examination. To be eligible for this arrangement, you must notify your instructor of your failure to take the test within a week of the missed midterm, and come up with a timeline acceptable to both for producing appropriate documentation for your absence. Please note that a student may NOT have 100% of their assessment based on the final examination. A student who has not completed a substantial portion of the term work normally shall not be admitted to the final examination. Refer to the Academic Policies if the final exam is missed.

The weights to compute the final grade are given in the following table.

Participation	5%
Assignments	40%
Midterm Exam	25%
Final Exam	30%

Learning Resources

Textbooks

- Bredon, Glen E. *Introduction to compact transformation groups.* Vol. 46. Academic press, 1972.

  This textbook is the **main resource** which the course will follow as is a classical introduction to the theory of Transformation Groups. However, much of the independent study comes from the lecture notes, the assigned activities and material found in the following suggested literature.

- tom Dieck, Tammo. *Transformation groups.* Vol. 8. Walter de Gruyter, 2011.
- Montgomery, D., Zippin, L. *Topological transformation groups.* Courier Dover Publications, 2018
- Husain, Taqdir. *Introduction to topological groups.* Courier Dover Publications, 2018.
- Pontryagin, L.S. *Topological Groups*; Princeton University Press: Princeton, NJ, USA, 1946.

Several of these textbooks can be found at the University library (physical copies) or an e-version (PDF) can be downloaded through the library website www.lib.uwo.ca

Online Resources

- The course website can be accessed through OWL www.owl.uwo.ca. Use your UWO username and password to log in.
- All important documents (syllabus, lecture materials, assignments, exam details, announcements, etc.) will be posted on the OWL website.
- Students are responsible for checking OWL for messages and announcements on a regular basis. This is the primary method by which information will be disseminated to all students in the class.
- The Forum tool is enabled on the OWL website. Please use this forum to post and respond to questions about course content (lecture, readings, practice questions, etc). Your instructor or teaching assistant will provide signed responses. *Courtesy and respect is mandatory for all forum postings and will be monitored by the instructor.*

Course Timetable

This is a tentative schedule for the topics that will be covered in the course. Some adjustments may be made as the course progresses, depending on the rate at which individual topics are covered. There are also listed relevant dates for assignments, projects and exams. Again, these dates are tentative and changes will always be announced during class and clearly documented through the OWL site.

Week	Dates	Course Topics	Important Dates and Reminders
1	Sep 7–Sep 11	Fundamentals of Topology and Group Theory (review)	
2	Sep 14–Sep 18	General Theory of Topological groups	Sep 18: Assignment 1 posted in OWL
3	Sep 21–Sep 25	Compact and locally compact Groups	
4	Sep 28–Oct 2	Group Actions and Orbit Spaces	Oct 2: Due date to submit assignment 1. Assignment 2 posted in OWL
5	Oct 5–Oct 9	Isotropy subgroups and fixed points	
6	Oct 12–Oct 16	Introduction to homotopy theory	Oct 16: Due date to submit assignment 2.
7	Oct 19–Oct 23	G-spaces and equivariant maps	Oct 23: Midterm Exam. It will cover topics from weeks 1-5.
8	Oct 26–Oct 30	Principal bundles and classifying spaces	Oct 30: Assignment 3 posted in OWL
9	Nov 2–Nov 6	Fiber bundles and the equivariant neighborhood theorem.	Fall Reading Week No Lectures
10	Nov 9–Nov 13	The Borel construction and introduction to equivariant theories.	Nov 13 : Due date to submit assignment 3 Assignment 4 posted in OWL.
11	Nov 16–Nov 20	Group actions on spheres and projective spaces	
12	Nov 23–Nov 27	Introduction to Differential manifolds	Nov 27: Due date to submit assignment 4.

13	Nov 30–Dec 4	Lie groups and Smooth actions	Last week of Lectures
Exam Period	Dec 7–Dec 11 Dec 14–Dec 18	December final exams period	Please do not book travel during this time

Lecture Policies

Please use the materials and resources provided on OWL and/or through class for your individual use during the course. Sharing or reproducing class materials online (for free or for profit) and/or sharing materials with individuals who are not taking the course is not acceptable without first receiving permission from the owner or creator of those resources/materials. Again, this is based on Intellectual Property rights.

Although you are welcome to use a computer/tablet during lectures and tutorials, you are expected to use the computer for scholastic purposes only, and refrain from engaging in any activities that may distract other students from learning. Please be respectful to your classmates and turn the sound off. If the professor receives complaints from other students regarding noise or other disruptive behaviour while using your electronic devices, your classroom privileges will be revoked. From time to time, the instructor may ask the class to turn off all computers, to facilitate learning or discussion of the material presented in a particular class. It is suggested to be mindful with your fellow classmates and limit the use of phones/iPods during the lectures and tutorials.

Ask your instructor and/or Teaching Assistant before you make an video/audio recording of class. This expectation provides basic respect for their privacy and personal safety, and is in keeping with Intellectual Property rights. If you would like to make video/audio recordings of our lecture sessions, please send me an email to arrange for an official permission.

Exam Policies

It is university policy that a regularly scheduled class (lecture or tutorial) takes precedence over tests and exams. Therefore, if another course schedules a test or exam that takes place during a lecture or tutorial of this class, the instructor for that course must accommodate you.

No electronic devices (calculators, laptops, tablets, phones, iPods, etc.) may be in your possession during tests and exams, even for timekeeping purposes.These devices must be left at home or with your bag/coat at the front of the room during an exam. They may not be at your test/exam desk or in your pocket. Any student found in possession of these prohibited devices will receive a mark of zero on the test or exam.

Inclusivity Statement

It is the intent that students from all diverse backgrounds and perspectives be wellserved by this course, that students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength and benefit. It is the intent to present materials and activities that are respectful of diversity: gender, sexuality, disability, age, socioeconomic status, ethnicity, race, and culture. Your suggestions are always encouraged and appreciated. The course is based on the premise of creating a learning environment for all students that supports a diversity of thoughts, perspectives and experiences, and honours student's identities. For example, If you have a name and/or set of pronouns that differ from those that appear in your official University records, please let me know!

Instructor Information

Name:	Sergio Chaves.
Office Hours:	Tuesday and Thursday from 4:00 to 5:00 pm in MC 109. Drop-in, no appointment necessary.
Email:	schavesr@uwo.ca

For *Teaching Assistants* information and office hours check the OWL course site.

Email Policies

Instructors and Teaching Assistants' email should only be used for administrative purposes. In order to maximize efficiency and to allow your instructors to respond to administrative concerns as quickly as possible, emails of the following nature will not be responded to: Questions that can be answered based on the information found in this course outline. and Requests for grade increases, extra assignments, makeup labs, etc.

If you email your instructor or designated teaching assistant, you must use your Western email address. Messages from a nonWestern account may be blocked by the universitys antispam system and will be ignored.

You can expect a response to an email Message or OWL Forum posting within about 48 hours during the Monday to Friday workweek. Note that Messages or Forum questions will generally not be answered in the 24-hour period before exams; this is meant to encourage proactive studying and help-seeking behaviour. Be sure to check OWL announcements and your UWO email on a regular basis for news and updates related to the course.

Policies and Support

Academic Policies

The website for Registrarial Services is http://www.registrar.uwo.ca. In accordance with policy, http://www.uwo.ca/its/identity/activatenonstudent.html,the centrally administered e-mail account provided to students will be considered the individuals official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at his/her official university address is attended to in a timely manner.

Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at this website: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf. If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or supporting documentation to the Academic Counselling Office of your home faculty as soon as possible. If you are a Science student, the Academic Counselling Office of the Faculty of Science is located in NCB 240, and can be contacted at scibmsac@uwo.ca.

For further information, please consult the universitys medical illness policy at http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_medical.pdf

If you miss the Final Exam, please contact your facultys Academic Counselling Office as soon as you are able to do so. They will assess your eligibility to write the Special Exam (the name given by the university to a makeup Final Exam).

You may also be eligible to write the Special Exam if you are in a Multiple Exam Situation (see http://www.registrar.uwo.ca/examinations/exam_schedule.html)

It is Faculty of Science policy that a student who chooses to write a test or exam deems themselves fit enough to do so, and the student must accept the mark obtained. Claims of medical, physical, or emotional distress after the fact will not be considered. Computer-marked, multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating.

All required papers may be subject to submission for textual similarity review to the commercial plagiarism detection software under license to the University for the detection of plagiarism. All papers submitted for such checking will be included as source documents in the reference database for the purpose of detecting plagiarism of papers subsequently submitted to the system. Use of the service is subject to the licensing agreement, currently between The University of Western Ontario and Turnitin.com (http://www.turnitin.com).

This course will use **CrowdMark**, an online collaborative grading and analytic platform. For information on their privacy policy, please visit their website, https://crowdmark.com/privacy. During tests/exams, proctors will inspect all personal belongings on your desk (and even your baseball cap if you are wearing one). If any items are discovered that are not permitted (e.g. any electronic device or other than a non-programmable calculator, or notes) they will be confiscated and the incident will be officially reported as an academic offence. Proctors have the discretion to move students between desks during the Tests or Exam periods.

Support Services

Please contact the course instructor if you require material in an alternate format or if you require any other arrangements to make this course more accessible to you. You may also wish to contact Services for Students with Disabilities (SSD) at 661-2111 ext. 82147 for any specific question regarding accommodation. The policy on Accommodation for Students with Disabilities can be found here: www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_disabilities.pdf. The policy on Accommodation for Religious Holidays can be found here: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_religious.pdf.

Learning-skills counsellors at the Student Development Centre (http://www.sdc.uwo.ca)are ready to help you improve your learning skills. They offer presentations on strategies for improving time management, multiple-choice exam preparation/writing, textbook reading, and more. Individual support is offered throughout the Fall/Winter terms in the drop-in Learning Help Centre, and year-round through individual counselling. Students who are in emotional/mental distress should refer to Mental Health at Western (http://www.uwo.ca/uwocom/mentalhealth/) for a complete list of options about how to obtain help. Additional student-run support services are offered by the USC, http://westernusc.ca/services.

Clicker Use in this Course

A clicker is a browser page or app opened on a personal WiFi device (e.g. a smartphone, tablet, or laptop). In class, instructors can ask a variety of structured questions to which you may respond by pressing the appropriate button on your device. Individual responses are collected and summarized ina graph at the front of the room. If the instructor chooses, individual responses may also be saved for future analysis.

Clicker Responsibility: The university is subscribed to and use clicker software produced by **iClicker** (https://www.iclicker.com/) because it is the company supported by Westerns technology services and is free to registered students. A student choosing to use a clicker will be responsible for (a) bringing their own device to use as a clicker, and (b) setting up their iClicker account correctly. Note that the course and instructor is not responsible(and therefore, no accommodation will be made)for WiFi failure. Clicker Academic Record. Your clicker use will be recorded in lecture and will become part of your academic record. As such, your clicker record will be afforded the same degree of security, confidentiality,and transparency that is customary for test marks, etc. Your clicker data will not be used for any non-academic or research purpose without your consent. For any research study in which you are invited to participate, you will be provided with a Letter of Information with an opportunity to give or withhold consent. Such research will not replace the usual end of term Student Questionnaire given by the University.

Academic Integrity: Use of a clicker associated with an identity other than your own is an academic offense. Granting permission for someone else to submit answers on your behalf in your absence is an academic offence. In a test, lab, lecture,or tutorial, possession of more than one clicker device, or one associated with the identity of another student, will be interpreted as intent to commit an academic offense and will be reported as such. This means that it will be considered an academic offense to answer a clicker question using an account other than your own.

Retention of Electronic Version of Course Outlines (Syllabi)

At the same time that course outlines/syllabi are posted on the appropriate Website, each Department must forward an electronic version of items 1-5 of each course outline (syllabus) to the Office of the Dean of the Faculty or College. By the fourth week after the start of term, the Deans Office will forward all of the collected outlines to Registrarial Services, where they will be maintained in electronic form in the faculty/staff extranet for a minimum of ten years after the completion of the course.(Final retention periods and disposition will be determined by the relevant records retention and disposition schedule approved by the President's Advisory Committee on University Records and Archive).

Acknowledgment of the Science Student Donation Fund:

Mathematics undergrad courses are supported by the Science Student Donation Fund. If you are a BSc or BMSc student registered in the Faculty of Science or Schulich School of Medicine and Dentistry, you pay the Science Student Donation Fee. This fee contributes to the Science Student Donation Fund, which is administered by the Science Students' Council (SSC). One or more grants from the Fund have allowed for the purchase of equipment integral to teaching this course. You may opt out of the Fee by the end of September of each academic year by completing the online form linked from the Faculty of Science's Academic Counselling site. For further information on the process of awarding grants from the Fund or how these grants have benefitted undergraduate education in this course, consult the Chair of the Department or email the Science Students' Council at ssc@uwo.ca.

UNIVERSIDAD COLEGIO MAYOR DE NUESTRA SEÑORA DEL ROSARIO
PROGRAMA DE ASIGNATURAS

FACULTAD: CIENCIAS NATURALES Y MATEMÁTICAS
PROGRAMA: ADMINISTRACIÓN DE EMPRESAS
ADMINISTRACIÓN DE NEGOCIOS INTERNACIONALES
ADMINISTRACIÓN EN LOGÍSTICA Y PRODUCCIÓN

Asignatura	Probabilidad
Código	73210012
Tipo de Saber	Básica ☒ Complementaria ☐ Formación Integral ☐
Tipo de Asignatura	Obligatoria ☒ Electiva ☐
Número de Créditos	Cuatro (4)
Prerrequisitos	Cálculo Diferencial e Integral
Correquisitos	
Período Académico	

JUSTIFICACIÓN Y UBICACIÓN EN EL PROGRAMA

La asignatura de Probabilidad es indispensable como herramienta para el planteamiento y solución de problemas del mundo empresarial y como complemento de otras áreas del saber, en especial, Administración, Negocios, Logística, Economía y Finanzas. Ofrece al investigador las bases conceptuales y practicas para la recolección y organización de los datos, su análisis e interpretación, y su aplicación en los negocios.

OBJETIVOS

Desarrollar habilidades para el planteamiento y solución de problemas prácticos, relacionados con:

- La estadística descriptiva y sus aplicaciones.
- Los fundamentos de la probabilidad y sus aplicaciones.
- Los modelos de probabilidad de variable aleatoria discreta y continua y sus aplicaciones.

Objetivos Específicos

- Entender la importancia de la estadística y sus aplicaciones en Administración y negocios.
- Identificar y diferenciar los conceptos de datos agrupados y no agrupados para el calculo de las medidas de estadística descriptiva.
- Reconocer y diferenciar las medidas de tendencia central, dispersión, forma y asociacion lineal.
- Reconocer y aplicar los conceptos básicos de probabilidad.
- Identificar y aplicar los conceptos de variable aleatoria y distribución de probabilidad.
- Diferenciar los conceptos y aplicaciones de los principales modelos de probabilidad discreta y continua.
- Desarrollar la capacidad analítica de los estudiantes para la interpretación de resultados y la aplicación a casos concretos de la Administración.
- Conocer la distribución muestral de los principales estadísticos

COMPETENCIAS A DESARROLLAR:

Al término del curso de Probabilidad el alumno será capaz de:

- Resolver problemas donde se involucren eventos con incertidumbre, aplicando los modelos analíticos apropiados.
- Obtener, analizar y representar gráficamente conjuntos de datos tomados de una situación real, haciendo síntesis de ellos, mediante descripciones numéricas.
- Aplicar los fundamentos de la teoría de la probabilidad en el cálculo de probabilidades de diferentes tipos de sucesos.
- Particularizar en el estudio de las distribuciones Binomial, Multinomial, Hipergeométrica, Poisson y Geométrica. Determinar el modelo matemático apropiado que deba aplicarse.
- Aplicar los conceptos de variable aleatoria continua, con base a situaciones reales o simuladas y establecer la correspondiente distribución de probabilidad continua, particularizar en el estudio de las distribuciones Uniforme, Exponencial y Normal. Aproximar la distribución Normal a la Binomial.
- Establecer un modelo matemático razonable que describa las relaciones entre las variables bajo estudio, y será capaz de medir el grado (magnitud) de dicha relación.

CONTENIDOS

Distribución de horas por temas

Tema	Horas
Tema 1. Las probabilidades y la estadística	2
Tema 2. La descripción de los datos	16
Tema 3: La probabilidad	12
Tema 4: Distribuciones de probabilidad discretas	12
Tema 5: Distribuciones de probabilidad continuas	14
Tema 6: Muestreo y distribuciones muestrales	8
Total	64

Contenidos por temas

Tema 1. Las probabilidades y la estadística
Aplicaciones a los problemas de Administración de Empresas. Los datos y sus fuentes. Elementos, variables y observaciones. Las escalas de medición. Los datos cualitativos y cuantitativos. Datos de sección transversal y de series de tiempo.

Tema 2. La descripción de los datos
Resumen de datos cualitativos: las distribuciones de frecuencias, frecuencias relativas y absolutas. Gráficas de barras y de pastel. El resumen de datos cuantitativos: las distribuciones de frecuencias y su representación a partir de los histogramas; distribuciones acumuladas y ojivas. Las medidas numéricas en la descripción de los datos. Medidas de localización: media, mediana, moda y los percentiles. Medidas de variabilidad: Rango, rango intercuartílico, varianza, desviación estándar, coeficiente de variación. Las medidas de forma de la distribución y la detección de observaciones atípicas: la forma de la distribución, las medidas de asimetría y curtosis. Los puntajes Z, su importancia. La detección de observaciones atípicas. Los diagramas de caja. El tratamiento de los datos agrupados: media y varianza.

Tema 3: La probabilidad
Experimentos y sucesos aleatorios. Operaciones con sucesos aleatorios. La probabilidad de un suceso: la noción frecuentista y subjetiva de probabilidad. La definición clásica y la aproximación por frecuencias de la probabilidad. Reglas de conteo, combinaciones y permutaciones, su uso en el cálculo de probabilidades. Reglas básicas para el cálculo de la probabilidad de sucesos compuestos. Los axiomas que definen la probabilidad. Probabilidad condicional, independencia de sucesos. Los teoremas de Bayes y de la Probabilidad Total.

Tema 4: Distribuciones de probabilidad discretas
Variables aleatorias discretas y continuas. Las distribuciones de probabilidad discretas, su representación a partir de la función de probabilidad. Valor esperado y varianza. Cálculo de probabilidades. Las distribuciones de Bernoulli, Binomial, Multinomial, Poisson, Hipergeométrica y Geométrica

Tema 5: Distribuciones de probabilidad continuas
Variables aleatorias continuas. Las distribuciones de probabilidad continuas, su representación a partir de la función de densidad de probabilidad. Cálculo de probabilidades utilizando la función de densidad, las probabilidades como áreas. Las distribuciones de probabilidad uniforme, normal y exponencial. Aproximación normal de las probabilidades binomiales.

Tema 6: Muestreo y distribuciones muestrales
Muestreo aleatorio simple en el caso de poblaciones finitas e infinitas. Otros métodos de muestreo: estratificado, sistemático, conglomerados, de conveniencia y subjetivo. Parámetros poblacionales y estimadores. Los estadísticos (estadígrafos). La estimación puntual y la distribución de probabilidad de los estimadores: las distribuciones muestrales. Distribución muestral de la media, proporciones y varianza, casos de una y dos poblaciones, su uso en el cálculo de probabilidades y la obtención de percentiles de las distribuciones. Las distribuciones t de student, ji-cuadrado y F de Fisher.

PRINCIPALES PRÁCTICAS PEDAGÓGICAS A UTILIZAR - METODOLOGÍA

- Talleres y tareas
- Monitorias
- Clases magistrales.

Al tener el semestre de estadística menos horas que este de probabilidades se consideró conveniente extender las nociones probabilísticas a las distribuciones muestrales, tema que usualmente se desarrolla en los programas de estadística. Tomando en consideración el nivel del programa de matemáticas que es pre-requisito de esta asignatura no se introducen las nociones probabilísticas para vectores aleatorios, sino que se partirá de la noción intuitiva de distribución muestral. El programa fue desarrollado siguiendo el texto básico.

FORMAS DE EVALUACION

Evaluación	Porcentaje
Parcial 1	20
Parcial 2	20
Parcial 3	20
Labor durante el curso	15
Examen final	25
Total	100

Para la organización de los parciales y evaluaciones frecuentes debe tener en cuenta las fechas límite para los cortes evaluativos que establece el calendario de la universidad. Esto significa que antes de esas fechas deben haberse realizado el parcial correspondiente y diferentes evaluaciones frecuentes que permitan conocer la situación de cada estudiante en la asignatura en ese momento del curso.

BIBLIOGRAFÍA

Básica:

Anderson D, Sweeny D, Williams T. Estadística para Administración y Economía. Cengage 2008

Complementaria:

- Newbold, Paul. Estadística para administración y economía. Pearson-Prentice Hall, 2008.
- Webster Allen L. Estadística aplicada a los negocios y a la economía. McGraw Hill, 2000
- Levine, Krehbiel and Berenson. Estadística para Administración.2006

Math 1600 Linear Algebra — Fall 2019

Tutorial 4 - Wednesday

Matrix Operations

1. Consider the following matrices:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix} \text{and } C = \begin{bmatrix} 2 & -5 \\ -2 & 4 \end{bmatrix}$$

   Calculate

   (a) $A + B^T$.

   (b) $A^T C$

   (c) $AB + C^T$

2. Let $A = \begin{bmatrix} 4 & 3a - b & 1 \\ 7 & -1 & 2 \\ 2a + 3b & 2 & 0 \end{bmatrix}$. For which values of a and b is A a symmetric matrix?

Inverse of a Matrix

1. Consider the linear system

$$\begin{cases} x - y + z &= -1 \\ -2y + z &= 2 \\ 2x + 3y &= 4 \end{cases}$$

   Write the system in the form $A\mathbf{x} = \mathbf{b}$. Find A^{-1} and verify that the vector $\mathbf{v} = A^{-1}\mathbf{b}$ is a solution to the system.

2. Let A be an invertible $n \times n$ matrix. Prove the following statements.

   (a) The matrix A^{-1} is invertible and $(A^{-1})^{-1} = A$.

   (b) For any non-zero scalar c, cA is invertible and $(cA)^{-1} = cA^{-1}$.

   (c) The matrix A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$.

   (d) For any positive integer n, A^n is invertible and $(A^n)^{-1} = (A^{-1})^n$.

   (e) For any invertible matrix B of the same size as A, $(AB)^{-1} = B^{-1}A^{-1}$.

True of False

Either prove the statement or give an example showing it is false.

1. If A and B are symmetric $n \times n$ matrices, then AB is also symmetric.
2. Let A, B, C $n \times n$ matrices. If A is an invertible matrix and $BA = CA$, then $B = C$.
3. Let A be a $n \times n$ matrix and v a vector in $\mathbb{R}^n$. If $Av = \mathbf{0}$ then $v = \mathbf{0}$
4. If A is $n \times n$ invertible matrix with entries in $\mathbb{Z}_2$, then $A^{-1} = A$.

Western Graduate& PostdoctoralStudies

April 5, 2019

Sergio,

Thank you for inviting me to attend your guest lecture on March 19, 2019, entitled *Euler's Formula and Platonic Solids.* Please feel free to use this unsolicited review as part of your teaching dossier, if you wish.

First, your lecture was well structured and had a clear organization. You began your lecture with a 'hook' to capture the audience's attention by posing a thought-provoking question, "could we identify how many polygons were in a soccer ball?" You proceeded by explaining the purpose behind the question and showing a slide of lecture outcomes. You also used a variety of tables and charts that summarized key points at the end of each section in the lecture. As well, you ended your presentation with an effective summary slide with the main takeaways from your lecture. This was very helpful to your audience as it emphasized the key points that may have been forgotten since they were discussed earlier in the lecture. As a minor suggestion, consider including more simplified explanations of the purpose behind the formula as this was, at first, unclear to some students.

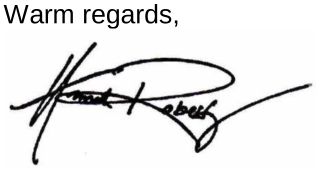
Secondly, you had an astute ability to explain complex concepts to your audience by deconstructing big ideas into smaller ones. When introducing the topic, you began by asking your students very simple questions, such as "what is a polygon?" and subsequently asking students to identify the correct polygons out of a variety of mixed shapes. This provided students with not only a clear definition but an opportunity to practice applying their knowledge. You continuously utilized interactive exercises with the class with handouts and gave the class adequate time to practice on their own. By taking up the worksheets after with the class, students were able to identify their own knowledge gaps and develop a clearer understanding of the content.

You also handled class disruptions very well. Some students entered the class late or received a phone call during your lecture, but you continued to teach the class as if there were no interruptions. This kept your audience attentive and fully focused on you as opposed to becoming distracted by other disruptions.

Finally, you utilized a variety of teaching methods throughout your lecture. Your use of PowerPoint provided clear and colourful visuals to follow along with your explanations of mathematical formulas. You also utilized the whiteboard to write out mathematical relationships and solve problems. Your handouts provided the class with the opportunity to practice the newly taught concepts on their own, and your overall enthusiasm kept the class very engaged.

Overall, this was a great lecture and good luck with your future teaching!

Warm regards,



Hannah Roberts
MSc. Candidate, Clinical Anatomy
Dept. of Anatomy and Cell Biology
Western University, Canada

April 5, 2019.

Dear Colleagues,

Hannah Roberts presented a lecture for *Histology 3309* on March 4, 2019. A peer observation occurred as part of the **Teaching Mentor Program** offered through the *Centre for Teaching and Learning* at Western University and feedback will be provided for this teaching lesson. The class session occurred in a laboratory where approximately 30 students attended to the lecture. A slides presentation was used in the lesson; but due to the laboratory layout, Hannah made use of a microphone and the students watched at any of the screens situated along the room.

- *Teaching Methods:* Hannah successfully uses instructional technology to enhance student learning. She communicates and presents material clearly and checks regularly for understanding. Hannah uses a variety of activities in class, as well as graphs, diagrams and images to facilitate explanation.

- *Organization and Preparedness:* Hannah demonstrates an accurate knowledge of the subject discussed during the lesson, this is also reflected into her use of voice speed, tone, volume and clarity. She reinforces learning goals consistently throughout the lesson, constantly reviews the concepts introduced and also compare the different notions that are being discussed along the way. Hannah incorporates adequate final concept checking activities and a final summary of the lesson highlighting the most important key concepts for the students to learn.

- *Student Engagement and Participation*: Hannah engages and maintains students in active learning by using interactive activities during the lesson. She introduces the topic with pretending to apply a “surprise quiz” which trigger a biological reaction which is related to the topic that will be discussed next. Hanna actively listens and pays attention to students’ questions and provides effective responses. However, there was neither enough room for checking students’ questions nor to add group activities.

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- *Additional observations*: Hannah establishes clear learning outcomes and constantly she reminds, restates, and reinforces the acquired knowledge to students. The room organization required Hannah to remain in a steady position and an appropriate movement throughout the room was hard to achieve. In particular, eye contact interaction with students is almost not present during this session.

Hannah provided an excellent performance in this session, she devoted a lot of personal effort and interest in student's learning which was clearly reflected in her enthusiastic, warm and supportive classroom environment.

Sergio Chaves Ramirez

Sincerely,

Sergio Chaves
PhD candidate. Department of Mathematics.

Universidad del Rosario

Universidad del Rosario

Problema del excursionista

Proyecto de Algebra lineal

María Paula Ordoñez – Lorena Pedraza – Sebastián Senosiain – Carol Vanegas

11 de noviembre de 2013

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Lorena Pedraza
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Carol Vanegas

Índice

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1. Preámbulo del problema

1.1. ¿En qué consiste?

El problema del excursionista o de la mochila es un problema simple de entender: hay una persona que tiene una mochila con una cierta capacidad y tiene que elegir qué elementos pondrá en ella. Cada uno de los elementos tiene un peso y aporta un beneficio. El objetivo de la persona es elegir los elementos que le permitan maximizar el beneficio sin excederse de la capacidad permitida.

Sea n objetos no fraccionables de pesos pi y beneficios bi. El peso máximo que puede llevar la mochila es C. Queremos llenar la mochila con objetos, tal que se maximice el beneficio.

1.2. ¿Por qué es importante?

Este problema es importante ya que responde a situaciones en donde se quiere obtener la mayor utilidad o beneficio posible, teniendo en cuenta que en estas situaciones se presentan ciertas restricciones. Esto se puede presentar con mayor frecuencia en campos como el industrial y económico, en donde generalmente hay una restricción presupuestaria o de recursos y en donde además se quiere maximizar el beneficio y utilidad.

El problema del excursionista tiene numerosas aplicaciones en la teoría como en la práctica. De este problema también surgen como sub problemas en varios algoritmos para problemas de optimización combinatoria complex y estos algoritmos beneficiaran cualquier mejoramiento en el campo de los problemas del excursionista.

Las aplicaciones prácticas de los problemas del excursionista no son limitados: asuma que N proyectos están disponibles para un inversionista, y que la ganancia obtenida de J proyecto es Pj , j=1,..., n. Este cuesta Wj para invertir en un proyecto J y solo C dólares están disponibles. Un plan de inversión óptimo puede ser encontrado resolviendo un *problema del excursionista 0-1*.

Otra aplicación aparece en un restaurante donde una persona tiene que escoger k opciones sin sobrepasar de C calorías, asumiendo que hay Ni platos para escoger entre K opciones i=1,…, k, y Wi,j es el valor nutritivo mientras que Pij es la clasificación diciendo que tan rico es el plato. Entonces una comida óptima seria encontrada resolviendo el *problema del excursionista opción-múltiple.*

El problema del empaquetado binario ha sido aplicado para cortar barras de hierro en un empresa, para así minimizar el número de barras usadas cada día, aquí Wj es el largo de cada pieza demandada, mientras que C es el largo de cada barra deliberada desde la fábrica. Aparte de estas simples ilustraciones debemos mencionar las aplicaciones más importantes: Problemas en las operaciones de carga, corte de inventario, control de presupuesto, y administración financiera.

Existen otras aplicaciones en distintas áreas y que tienen distintos tipos de recurso limitante. Algunos ejemplos son:

- **Selección de oportunidades de inversión**: presupuesto como limitante. Un estudio interesante se puede encontrar que analiza situaciones en las cuales los elementos que se pueden incluir en la mochila se van recibiendo en forma continua y se debe tomar una decisión de que elementos elegir sin haber recibido todos (Problema de la secretaria)
- **Desperdiciar la menor cantidad de tela**: material como limitante.
- **Aprovechar al máximo el uso de máquinas**: tiempo como limitante

1.3. Trasfondo histórico

El problema del excursionista es uno de los 21 problemas *NP*-complejos de Richard Karp, establecidos por el informático teórico en un famoso artículo de 1972.

Los problemas de la mochila han sido intensamente estudiados desde el trabajo pionero de Dantzig, por su inmediata aplicación en la industria y administración financiera, pero más pronunciada por razones teóricas.

En esas aplicaciones se necesita resolver el problema del excursionista cada vez que una función es derivada demandando soluciones técnicas extremadamente rápidas. Diferentes tipos de problemas del excursionista ocurren, dependiendo de la distribución del objeto y la mochila: en el *0-1* cada objeto debe ser escogido como máximo una vez, mientras que en el *problema de la mochila delimitada* tenemos que delimitar una cantidad de cada tipo de objeto. El *problema de elección múltiple* ocurre cuando los objetos deben ser escogidos desde clases disjuntas y si hay varias mochilas llenas simultáneamente adquirimos el *problema de la mochila múltiple*. La *forma general de restricciones múltiples*, el cual es básicamente programación lineal con coeficientes positivos. Todos los problemas del excursionista pertenecen a la familia de los *problemas complejos NP,* que significa que son poco probables que se pueda alguna vez idear algoritmos poligonales para estos problemas.

Como estos problemas del excursionista son *NP* no necesitamos saber alguna otra solución técnica exacta, que una enumeración del espacio de la solución (posiblemente completa). Estas son algunas de las técnicas usadas:

- Programación entera (ramificación y acotamiento)
- Programación dinámica
- Reprocesamiento

1.4. Referencias de aplicaciones

En la vida real, se utiliza para modelar diferentes situaciones:

1) En los sistemas de apoyo a las fianzas: para encontrar el mejor equilibrio entre el capital y rendimiento financiero.

2) En la carga del barco o avión: todo el equipaje debe ser llevado, sin ser sobrecargado.

3) En el corte de materiales: para minimizar las caídas.

Entre otras...

2. Planteamiento general del problema

2.1. Planteamiento general

En el problema tipo mochila se deben identificar los siguientes elementos:

n: número de objetos entre los que se puede elegir

ai: peso del objeto “i”, y este objeto i-esimo es aquel que hace referencia a un objeto cualquiera que se pueda incluir dentro de la mochila

ci: costo de escoger el objeto i en tanto que este va a ocupar un espacio muy importante dentro de la mochila

bi: utilidad o beneficio que proporciona cada objeto, de nuevo se hace referencia al objeto i-ésimo.

p: capacidad de la mochila, es decir el presupuesto máximo del que se dispone.

En este problema se cuenta con sólo una restricción, es decir la máxima capacidad de la mochila. De esta manera la suma del espacio multiplicado por los elementos que se van a introducir a la maleta, no podrán ser mayores a la capacidad máxima de ésta. Este modelo se considera binario ya que en esta situación se puede tomar la decisión de llevar o no llevar el objeto, es decir solo son dos decisiones.

Por otra parte la restricción que nos surge viene dada por la capacidad máxima de la mochila, de tal forma que para este problema tenemos:

$$\sum_{i=1}^{n} c_i x_i \leq P$$

Por último, el objetivo del problema es maximizar el beneficio:

$$\text{Maximizar} \sum_{i=1}^{n} b_i x_i$$

En conclusión la solución del problema es:

$$\text{Maximizar} \sum_{i=1}^{n} b_i x_i$$

$$\text{s.a} \qquad \sum_{i=1}^{n} c_i x_i \leq P$$

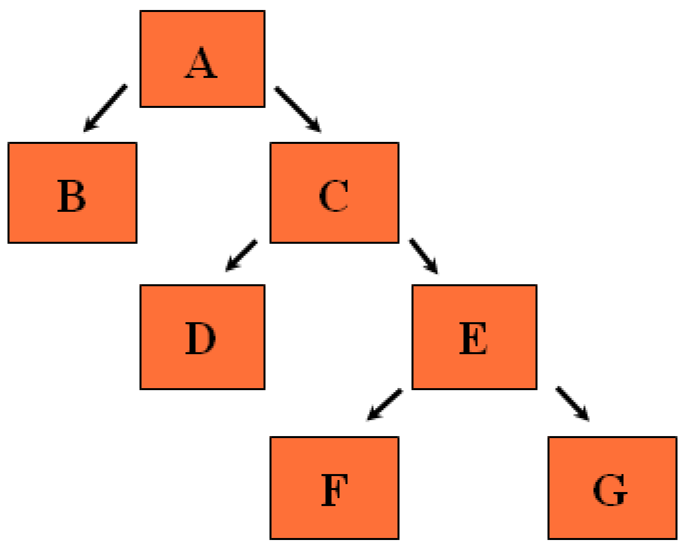
$$x_i \in \{0,1\}, \quad i = 1,\ldots,n$$

2.2. Solución general del problema

Los métodos que se exploraran para este método son los siguientes:

2.2.1. Ramificación y acotamiento

El método de ramificación y acotamiento o también conocido método de branch and bound, es un método en el cual se interpreta como un árbol de soluciones, donde en cada ramificación posible va arrojando diferentes resultados, y paso a paso cortando las ramas, llegaríamos a una rama o solución óptima.



Como descripción general, podríamos tener un objetivo en el problema y podría ser encontrar el valor mínimo de una función donde fijamos x rangos sobre un determinado conjunto de S de posibles soluciones. Para poder realizar la solución de un problema por este método, se es necesario dos cosas, la primera es un "procedimiento" de expansión, dado un conjunto S, devuelve conjuntos más pequeños tales como S1, S2,...,Sn, donde hay un mínimo f(k) sobre S (V1,V2,...) donde cada Vi es un mínimo de F(k). Como hablábamos anteriormente, para aplicar este método son necesarios dos pasos, el primero (el acabado de mencionar), es relacionado con la ramificación, y se tiene un algoritmo que dice: si la menor rama para algún árbol nodo A, es mayor que la rama padre para otro nodo B, entonces A tiene que ser eliminada de la búsqueda u obtención de la rama solución o rama optima, este segundo paso es asociado con el acotamiento.

2.2.2. Programación dinámica

La técnica de programación dinámica se emplea más que todo para resolver problemas de optimización y permite resolver problemas por medio de una secuencia de decisiones en donde sólo al final se sabe cuál es la mejor decisión. Este método está basado en el principio de optimalidad de bellman: "Cualquier sub-secuencia de decisiones de una secuencia óptima de decisiones que resuelve un problema también debe ser óptima respecto al sub-problema que resuelve." (J.Campos, 2009)

Para el problema de interés es decir el problema de la mochila hay n *elementos fraccionables* y una mochila, el objeto i tiene el peso pi y una fracción xi donde (0≤xi≤1) del elemento i que produce un beneficio bixi.
Lo que se quiere hacer es llenar la mochila la cual tiene una capacidad C con el fin de maximizar el beneficio. Esta capacidad viene a ser la restricción de recursos que se tiene. Luego la función objetivo a maximizar es la sumatoria del producto del beneficio y la fracción del elemento sujeta a la sumatoria del producto del peso y la fracción de los elementos la cual debe ser menor o igual a la capacidad con la que cuenta la mochila. Adicionalmente se debe tener en cuenta que el valor de los productos y los beneficios debe ser positivo.

$$\text{maximizar} \quad \sum_{1 \le i \le n} b_i x_i$$

$$\text{sujeto a} \quad \sum_{1 \le i \le n} p_i x_i \le C$$

$$\text{con} \quad 0 \le x_i \le 1,\ b_i > 0,\ p_i > 0,\ 1 \le i \le n$$

Hay un aspecto a tener en cuenta importante y es la variante de la mochila 0,1. Lo que se quiere tener en cuenta en este punto es que xi solo toma valores de cero a uno, lo que esto quiere decir es que el objeto se incluye o se deja fuera de la mochila. Los pesos pi son números naturales y los beneficios son reales no negativos.

2.3. Generalidad y particularidad

Independientemente del método que utilicemos para resolver este problema, bien sea el método simplex, el método de ramificación y acotamiento, el método de programación dinámica, o cualquier otro tipo de método que exista, se resolverá de forma general y específica, puede tener las restricciones que el problema demande, dependiendo de la situación en la cual se vaya a necesitar en el problema y de igual manera, la resolución de éste va a ser completamente diferente, todo depende del método a implementar. El tipo de procedimiento que exista, depende del método que utilicemos, el procedimiento varía, pero la respuesta óptima siempre debe ser la misma (si el problema tiene las mismas restricciones y los mismos datos para cualquier caso).

3. Exposición particular del problema

3.1.

Ejemplo 1

Una empresa dispone de un millón de euros para invertir en nuevos proyectos. En concreto dispone de 3 nuevos proyectos posibles. En la siguiente taba aparece el coste que supone cada uno de ellos, así como el beneficio que se espera de su realización. La empresa desea saber en cual debe invertir si quiere maximizar si beneficio esperado sin superar su presupuesto (cantidades en miles de euros):

	Coste inversión	Beneficio esperado
Proyecto 1	500	1.750
Proyecto 2	600	2.000
Proyecto 3	400	1.500
Proyecto 4	550	1.900

Para resolverlo, se llama xi a una variable binaria que toma el valor 1 si se elige el proyecto i (i=1, 2, 3, 4) y cero en caso contrario.

En este problema se encuentra como restricción solo la presupuestaria luego la formulación del mismo es como “problema de la mochila”

Maximizar: $1750\,x1 + 2000\,x2 + 1500\,x3 + 1900\,x4$

Sujeta a: $500\,X1 + 600\,x2 + 400\,x3 + 550\,x4 \leq 1000$

Con $x1, x2, x3, x4 \in \{0,1\}$

Resolviéndolo se obtiene que la mejor opción es elegir los proyectos 2 y 3 alcanzándose un beneficio esperado de 3.500.000 €.

Ejemplo 2

El club de Baloncesto Unicaja de Málaga quiere contratar un jugador nuevo; para ello, ha sondeado el mercado y ha encontrado a 5 jugadores que pueden adaptarse a lo requerido por el entrenador. Para reforzar el equipo en Unicaja dispone de un presupuesto máximo de 50.000 €/ mes. En la siguiente tabla aparece una relación de los candidatos a ser fichados junto con su aportación esperada y el sueldo que recibirían:

	SUELDO	APORTACION
Jugador 1	€50.000	15
Jugador 2	€25.200	8
Jugador 3	€36.000	15
Jugador 4	€47.000	17
Jugador 5	€12.000	7

Como puede apreciarse en este caso, estamos aplicando el problema de la mochila a una situación de índole económica. Nuestra intención es elegir los mejores jugadores, es decir aquellos cuya aportación es mayor y proporcionan una mayoría utilidad para el Unicaja, optimizando también el desembolso que conlleva una nueva contratación. No debemos olvidar la restricción de 50.000 €.

Si llamamos Xi al jugador “i” el problema a resolver es:

Maximizar $15x_1 + 8x_2 + 15x_3 + 17x_4 + 7x_5$

- **Sujeto a:** $50.000x_1 + 25.200x_2 + 36.000x_3 + 47.000x_4 + 12.000x_5 \leq 50.000$

  $x_1, x_2, x_3, x_4, x_5 \geq 0$

Donde la función objetivo es la suma de las utilidades que reporta cada jugador y representa, por lo tanto, la utilidad que percibirá Unicaja en función de la combinación de jugadores que elija, puesto que se trata de la utilidad (en este caso, estará medida por el número de partidos que el jugador haga ganar o en los que tenga un peso importante) al club de baloncesto le interesara que sea lo mayor posible. De ahí que el objetivo sea maximizar la función.

En cuanto la restricción, es la restricción presupuestaria del equipo, es decir, son los 50.000 € mensuales de los que puede disponer Unicaja para remunerar a sus nuevos jugadores.

Para resolverlo, utilizaremos las dos formas vistas. Primero, utilizando el planteamiento "lógico", escogeremos como criterio el ratio "Aportación/sueldo", ya que tenemos en cuenta ambos factores en la decisión: cuando más alto sea este ratio, preferible será contratar a ese jugador. Reconsideraremos el sueldo, dividiéndolo por 1.000 para hacer el ratio más operativo:

	SUELDO	APORTACION	RATIO=A/S
Jugador 1	50	15	0,3000
Jugador 2	25,2	8	0,3175
Jugador 3	36	15	0,4167
Jugador 4	47	17	0,3617
Jugador 5	12	7	0,5833

Como hemos dicho, escogeremos aquellos jugadores con mejor ratio hasta agotar el presupuesto:

- **Jugador 5**: ratio= 0,58333= 12.000 €
- **Jugador 3**: ratio= 0,41666= 36.000 €

Como el total disponible era de 50.000 € y tenemos acumulado 48.000 €, no hay más jugadores cuyo sueldo pueda entrar en presupuesto, así que este es nuestro resultado definitivo por este método.

Por otra parte, resolviendo obtenemos la misma solución, contratando a los jugadores 3 y 5. La utilidad para el equipo de los jugadores nuevo al equipo de 22.

3.2. Solución del problema real

El entrenador del equipo América de México quiere hacer una reestructuración total de equipo, para esto se ha hecho un examen detallado determinando las posiciones que se pueden reforzar. El entrenador desea reforzar las posiciones que le generen más beneficios al equipo. El equipo tiene un presupuesto de $700.000.00 (pesos mexicanos) mensuales y los aportes de cada posición son los siguientes:

Posición	Gasto Mensual	Aportación
Defensa	$100,000.00	2
Lateral	$200,000.00	4
Medio	$200,000.00	5
Delantero	$400,000.00	7

Análisis de la situación: De acuerdo a la descripción se puede ver cómo en un ámbito como el deportivo este problema también es de gran importancia, para este caso particular, se requiere escoger los jugadores y posiciones que generen mayor beneficio y utilidad para el equipo, para ello hay ciertas restricciones de gasto mensual y el aporte que cada posición brinda a beneficio del equipo. El problema de la mochila ayuda a tomar la decisión más acertada para hacer los refuerzos que brinden mayor beneficio al equipo.

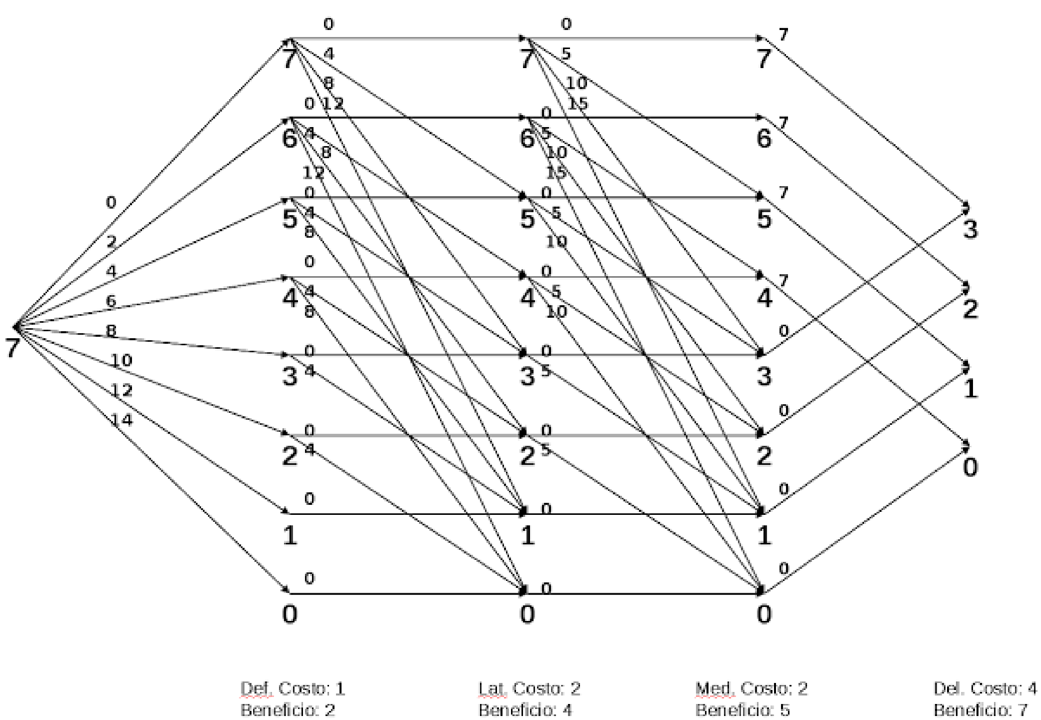
María Paula Ordoñez
Lorena Pedraza
Sebastián Senosiain
Carol Vanegas

Solución del problema

Para resolver este problema, utilizaremos programación dinámica. Para ello es necesario determinar todos sus elementos en este problema.

- **Etapas:** Posiciones disponibles para contratar jugadores (5 etapas)
- **Estados**: Dinero disponible para contratar jugadores de la posición t
- **Decisiones**: Jugadores a contratar de la posición t
- **Formula de recurrencia**: f*t (i , j) = dij + f*t+1(i)
- **Condición en la frontera**: f*5(i) = 0
- **Principio de optimalidad**: f*t (i) = max ft (i , j)

Ahora podemos determinar la gráfica para visualizar el problema y resolverlo de manera más sencilla. Los nodos representan el presupuesto disponible para cada posición, las aristas el beneficio que obtendrá el equipo.



María Paula Ordoñez
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Aplicación del Método de Solución

Para resolver el modelo nos valemos de las tablas, aplicando el principio de optimalidad y la condición en la frontera.

t=4

i \ j	3	2	1	0	$f^*_4(i)$	x_t
7	7	-	-	-	7	1
6	-	7	-	-	7	1
5	-	-	7	-	7	1
4	-	-	-	7	7	1
3	0	-	-	-	0	0
2	-	0	-	-	0	0
1	-	-	0	-	0	0
0	-	-	-	0	0	0

t=3

i \ j	7	6	5	4	3	2	1	0	$f^*_3(i)$	x_t
7	7	-	12	-	10	-	15	-	15	3
6	-	7	-	12	-	10	-	15	15	3
5	-	-	7	-	5	-	10	-	10	2
4	-	-	-	7	-	5	-	10	10	2
3	-	-	-	-	0	-	5	-	5	1
2	-	-	-	-	-	0	-	5	5	1
1	-	-	-	-	-	-	0	-	0	0
0	-	-	-	-	-	-	-	0	0	0

t=2

i \ j	7	6	5	4	3	2	1	0	$f^*_2(i)$	x_t
7	15	-	14	-	13	-	12	-	15	0
6	-	15	-	14	-	13	-	12	15	0
5	-	-	10	-	9	-	8	-	10	0
4	-	-	-	10	-	9	-	8	10	0
3	-	-	-	-	5	-	4	-	5	0
2	-	-	-	-	-	5	-	4	5	0
1	-	-	-	-	-	-	0	-	0	0
0	-	-	-	-	-	-	-	0	0	0

t=1

i \ j	7	6	5	4	3	2	1	0	$f^*_1(i)$	x_t
7	15	17	14	16	13	15	12	14	17	1

Interpretación de Resultados

Aportación al equipo de las contrataciones: 17

Posición	Jugadores a contratar
Defensa	1
Lateral	0
Medio	3
Delantero	0

Se debe contratar solo un jugador que refuerce la defensa y tres jugadores que refuercen la media cancha. Con esto se gastan los $700,000.00 disponibles cada mes pero se obtiene una aportación total de 17 puntos al equipo con dichas contrataciones.

Conclusiones del problema

Con este ejemplo podemos notar que es posible resolver el problema tipo mochila con programación dinámica, este método permite repetir un mismo artículo varias veces, el inconveniente con este tipo de modelos es la gran cantidad de nodos y aristas que se generan para modelos más grandes; en este caso, con solo cuatro objetos, obtuvimos 26 nodos y más de 40 aristas. Esto es algo que debemos tomar en cuenta a la hora de determinar el método de solución para este tipo de problemas.

3.3. Programa (C++)

El programa debe ser descargado para poder ser visualizado.

4. Conclusiones

Después de observar y analizar el problema del excursionista o problema de la mochila, podemos concluir que no solo sirve para resolver este tipo de problemas, sino que también nos puede ayudar para la vida cotidiana en donde nos encontremos con algunas situaciones (sin importa el ámbito), en las que la situación cuente con cierto número de restricciones y además se quiera obtener una utilidad o un beneficio óptimo.

De acuerdo a la investigación previamente realizada, también se puede evidenciar como ciertas empresas del ámbito financiero logran hacer que su presupuesto se equilibre con los gastos de la misma, utilizando el método del excursionista puesto que se presta para este ámbito.

Se puede observar que la programación dinámica es un método que reduce el tiempo de ejecución de un algoritmo, dando soluciones optimas de sub-problemas para encontrar la solución. En su conjunto se debe tener en cuenta que el problema de la mochila es un problema NP- que al incrementar el número de objetos las restricciones varían y se requiere la implementación de métodos algorítmicos que lleven a una respuesta. La técnica que se ha implementado da como resultado la solución óptima.

Las diferentes metodologías desarrolladas para la solución de problemas de este tipo tienen principales factores en común, y todas buscan llegar a la misma respuesta en donde aunque distintos elementos brinden un beneficio, solo los elementos que más beneficio brinden en conjunto serán los elementos respuesta. Este problema es de fácil contextualización por lo que así mismo es de fácil implementación para muchas situaciones en donde hay restricciones de recursos o presupuestarias de manera que estas metodologías buscan principalmente facilitar el uso de recursos de muchos tipos, en cuanto a problemas de carácter binario.

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THE CHALLENGE OF TEACHING MATHEMATICAL PROOFS TO UNDERGRADUATE STUDENTS

CAPSTONE PROJECT
ADVANCED TEACHING PROGRAM

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ABSTRACT

Teaching mathematical proofs for undergraduate students is a challenge for both instructors and students. Proof is an essential part on learning as it develops reasoning skills, improves problem solving aptitudes and it is also a fundamental basis for future researchers. This reflection is focused on studying the difficulties experienced in the learning and teaching process of formal mathematical proofs, and discuss whether the traditional "definition-theorem-example" lecture method is the most appropriate for the students to success in mathematical courses.

1 Introduction

The role of proving in mathematical courses is not only important for success in the course, but also for developing abstract skills and deductive reasoning; which is essential for many future professionals in any area. Indeed, an important aspect in proving is that this process provides certainty for the results together with the solution of a given problem [Hanna(1995)]; moreover, it provides opportunity to justify not only the correctness of reasoning but also to convince others about its validity [Ko(2010)]. In short, a proof is a mean to check whether an explanation and its reasons are correct or incorrect as single and a whole structure. The learning of how to make deductive reasoning, and translate it into valid and acceptable explanations is different for every student; in fact, several factors affect university students and cause a bias towards avoiding any proof-related activity during undergraduate courses. For example, the lack of problems and approaches to proving starts from middle and high-school. On the other hand, large classrooms where multiple choice test are applied, the interaction of the students with abstract proofs is barely discussed. Therefore, these skills becomes difficult to obtain for many students. This is a challenge that theoretical disciplines encounter, and in particular, the proving skills developed in Mathematics are non-empirical; more precisely, they require advanced theoretical knowledge which is used in a rigorous way in mathematics and their ramifications. For example, the mathematical theorems are statement which must be proven rigorously through other previous theorems, axioms or accepted statements; these theorems build the science of the mathematics and they lead to applications in several different areas. In this reflection, the difficulties presented in the process for both students and instructors when facing mathematical proofs in undergraduate courses will be mainly addressed, and also possible solutions for future instructors can consider to overcome these difficulties will be discussed.

2 Annotated Bibliography

2.1 The Difficulties Experienced in Teaching Proof to Prospective Mathematics Teachers: Academician Views [Güler(2016)]

This study firstly identifies the main difficulties that both students and teachers face when mathematical proofs are including in the lectures. Throughout a study performed across 7 universities, 15 instructors and 30 courses during a period of one year, the main difficulties found are the following.

- Students believe that mathematical formalities were proved years ago and it is *unnecessary* to review proofs during the lectures.
- Students face a sudden encounter with mathematical proofs at the university level, as this is mainly ignored before university education.
- Using complex concepts and definitions, as well as the process of understanding a proof is not commonly taught.
- Instructors use superficial methods to asses students' understanding of mathematical proofs.

The difficulties are classified into three main categories.

- **Prior knowledge before university level**. This is caused mainly because in middle and high-school mathematical thinking is neither discussed nor assessed. Also, standardized tests and university entrance exams focus more on computational problems than deductive thinking. Finally, students prepare fora answering short-answer and multiple-choice type questions.
- **Understanding proof methodologies**. Students encounter a lot of difficulties in starting a proof, they also find hard how to connect ideas and using the appropriate deductive steps. Instead of understanding and ask about the logic behind the reasoning, students memorize the proof which do not allow them to develop deductive thinking.
- **Bias against proofs**. Students are not motivated and do not regard proving as an enjoyable process like solving a problem, but rather as case approached with more fear. They feel anxiety against proving-related questions, and they have a perception that proving is a very complex but unnecessary process.

Among possible solutions to overcome these difficulties it is proposed that instructor's awareness about them, the importance and meaning of proving should be raised especially in mathematics courses. More precisely, involve not only instructors, but education programs and course coordinators to address, discuss and prevent proving from being regarded as an unnecessary activity. Furthermore, justify and appraise the benefits of acquiring and developing mathematical skills will help to decrease the bias towards proofs, indeed, for any future professional, analytical and problem solving skills, as different point of view to face a problem are indispensable skills that can be build and reinforced in mathematical courses.

2.2 The Meaning of Proofs in Mathematical Education [Reid(2005)]

This study focus on the inner concept of proof, the purpose of teaching mathematical proofs and the kind of reasoning that proving involves. The concept of a proof depends on the context. A formal logical deductive proof should be different to a teaching proof; in fact, the first one is more rigorous and it inherits a status of *a priori universal validity*. On the other hand, teaching proof should be a quasi-empirical method: Instead of showing the students *what* is the proof, it should be an analytical process that goes from the exemplification to the abstraction, and students should learn *how* is it proved and *why* the proof is made in that particular way.

Regarding the purpose of teaching mathematical proofs, this study suggest that this will develop on students the ability to think critically in several domains of explanation. It also has a critical thinking purpose, suggesting that showing students the limits of mathematics would make them more critical of the use of numerical arguments in other domains. Finally, teaching proofs should not be strictly deductive as the evolved reasoning of the instructor has became, but involving empirical and a combination of other kind of reasoning will boost student acceptance and understanding of proofs. The goal is to provide a structure of proving as follows: verification, exploration, explanation, systematization, communication and social acceptance.

2.3 Conjecturing, generalizing and justifying: Building theory around teacher knowledge of proving [Lesseig(2016)]

This study is based on some of the practices developed in the process of proving, these practices consists of *conjecturing, generalizing and justifying*. The step of conjecturing is the ability to develop statements that could be true, generalizing is to extend hypothesis to valid statements or claims using mathematical language and abstractions, and finally justifying is the act of developing arguments to demonstrate the truth (or falsehood) of a claim using mathematical forms of reasoning. To teach students to develop knowledge and practice on these crucial steps to successfully proving, the study found that the lack of knowledge required of teachers to adequately support students' understanding of and engagement in these practices. Recognizing the importance of promoting these disciplinary practices in classroom communities allows that students at all levels can engage, motivate and obtain opportunities in a supportive environment.

There is a research trend called mathematical knowledge needed for teaching (MKT) about how mathematical knowledge is required and used to meet the demands of classroom teaching and to identify mathematical e and pedagogical content knowledge central to the work of teaching proof. In this study, it is defined two subject matter for teaching proof; namely, the *Common Content Knowledge (CCK)* as the ability to solve mathematical problems; in other words, to construct proofs for oneself, and the *Specialized Content Knowledge (SCK)* as the knowledge necessary to support teaching this mathematics to new learners, and it is a useful way to conceptualize the range of job-specific knowledge required.

3 Personal Reflection

The teaching and learning challenge that I decided to focus in this study is the challenge of teaching mathematical proofs to undergraduate students. As the three main references suggest, learning to do mathematical proofs for students is important not only for the success during the courses and get higher grades, but also for them as future professional in any area. This knowledge generates appropriate tools to face, model and solve problems; as well as it enhances critical and deductive thinking to support and defend conjectures or hypothesis. Finally, for those that purse a research career, providing valid proofs is an essential skill that prospective students must develop.

This is an unique challenge for the discipline given that courses in mathematics provide both the abstract and applicable setting to broad theories. As mathematics is one (or the most) abstract science, to plant an analytical and general deductive thinking on students this should be the place to start. To fight against the low-rate of skilled professionals in STEM careers, it is necessary to both teach and motivate students to engage and enjoy proving.

In the annotated bibliography, investigators have developed studies for the learning of proofs from the student perspective as well as the teaching and the difficulties experienced in this process by both students and instructors. However, the more general difficulties are presented to the students due are a lack of exposition to proofs before university education and a focus in multiple choice questions and short answers in courses of first year. This is translated into the fact that many students are not able to develop abstract thinking and changing the bias towards the proofs even before accessing to them. Given that a primary instructional goal is to support students' progression toward deductive proof, conceptualizing proving activity in this way has great educative value. A major aim in mathematics instruction is to move students from inductive justifications toward deductive proof. Teachers productively used examples to make, test, and refine conjectures and then began to develop mathematical arguments to justify their conclusions. Teacher knowledge is a first step to support students in the fundamental mathematical practices of conjecturing, generalizing and justifying; however, there should be a clear distinction for instructors between formal proof, those presented in a research environment, and teaching proofs, the approach that should be taken in the classroom.

Current and future mathematics instructors should take in account this challenge and realize the severity of the same, considering that traditional teaching and assessing methods might not be the most appropriate way for students to acquire deductive and reasoning skills. Also, there should be an awareness of this problem in a broader level; namely, not only students in STEM programs and instructors from mathematics courses, but also interdisciplinary discussion will help to attack these challenges. For example, in most of all of the universities there is no course that teach about proofs for a general audience; more precisely, such a course is intended for students pursing a Mathematics mayor, and for the remaining students this knowledge is barely offered on the different courses. For these reasons the support of the University and instructors is essential to develop appropriate tools to address all these challenges.

For future teaching, it is important to leave the traditional teaching and assessment methods. Implementing active learning activities that engage students into proofs by leading them to conjecture, analyze and proof important mathematical facts that appears in any level in mathematics. Mathematics is a discipline that is getting late into transitioning and implementing modern teaching techniques using any of the several available technological tools that helps students to learn and the instructor to *make mathematics not boring*; but starting for one-self and the colleagues around is the first step towards a huge change. Students tend to think that mathematics is a difficult subject, have negative feelings towards the subject and it is only suitable for genius; in fact, one does not need to be a genius to enjoy and do mathematics, and there is no such a thing as *"I am not a math person"*[1].

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