THE CHALLENGE OF TEACHING MATHEMATICAL PROOFS TO UNDERGRADUATE STUDENTS

CAPSTONE PROJECT

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ABSTRACT

Teaching mathematical proofs to undergraduate students is a challenge for both instructors and students. Proof is an essential part on learning as it develops reasoning skills, improves problem solving aptitudes and it is also a fundamental basis for future researchers. This reflection is focused on studying the difficulties experienced in the learning and teaching formal mathematical proofs, and discuss whether the traditional "definition-theorem-example" lecture method is the most appropriate for the students to succeed in mathematical courses.

1 Introduction

The role of proving in mathematical courses is not only important for succeeding in the course, but also for developing abstract skills and deductive reasoning; which is essential for many future professionals in any area. Indeed, an important aspect in proving is that this process provides certainty for the results together with the solution of a given problem [Hanna(1995)]; moreover, it provides opportunity to justify not only the correctness of reasoning but also to convince others about its validity [Ko(2010)]. In short, a proof is a mean to check whether an explanation and its reasons are correct or incorrect as single and a whole structure. Learning how to make deductive reasoning and translate it into valid and acceptable explanations is different for every student; in fact, several factors affect university students and cause a bias towards avoiding any proof-related activity during undergraduate courses. For example, the lack of problems and approaches to proving starts from middle and high-school. On the other hand, large classrooms where multiple choice test are applied, the interaction of the students with abstract proofs is barely discussed. Therefore, these skills become difficult to obtain for many students; this is a challenge that theoretical disciplines encounter, and in particular, proving skills developed in Mathematics are non-empirical; more precisely, they require advanced theoretical knowledge which is used in a rigorous way in mathematics and their ramifications. For example, the mathematical theorems are statement which must be proven rigorously through other previous theorems, axioms or accepted statements; these theorems build the science of mathematics and they lead to applications in several different areas. In this reflection, the difficulties presented in the process for both students and instructors when facing mathematical proofs in undergraduate courses will be mainly addressed, and also possible solutions that future instructors can consider to overcome these difficulties will be discussed.

2 Annotated Bibliography

2.1 The Difficulties Experienced in Teaching Proof to Prospective Mathematics Teachers: Academician Views [Güler(2016)]

This study firstly identifies the main difficulties that both students and teachers face when mathematical proofs are including in the lectures. Throughout a study performed across 7 universities, 15 instructors and 30 courses during a period of one year, the main found difficult are the following.

- Students believe that mathematical formalities were proved years ago and it is *unnecessary* to review proofs during the lectures.
- Students face a sudden encounter with mathematical proofs at the university level, as this is mainly ignored before university education.
- Using complex concepts and definitions, as well as the process of understanding a proof is not commonly taught.
- Instructors use superficial methods to asses students' understanding of mathematical proofs.

The difficulties are classified into three main categories.

- **Prior knowledge before university level**. This is mainly caused because in middle and high-school mathematical thinking is neither discussed nor assessed. Also, standardized tests and university entrance exams focus more on computational problems than deductive thinking. Finally, students prepare for answering short-answer and multiple-choice type-questions.
- **Understanding proof methodologies**. Students encounter a lot of difficulties in starting a proof, they also find hard how to connect ideas and using the appropriate deductive steps. Instead of understanding and ask about the logic behind the reasoning, students memorize the proof which do not allow them to develop deductive thinking.
- **Bias against proofs**. Students are not motivated and do not regard proving as an enjoyable process like solving a problem, but rather as case approached with more fear. They feel anxiety against proving-related questions, and they have a perception that proving is a very complex but unnecessary process.

Among possible solutions to overcome these difficulties it is proposed that instructor's awareness about them, the importance and meaning of proving should be raised especially in mathematics courses. More precisely, involve not only instructors, but education programs and course coordinators to address, discuss and prevent proving from being regarded as an unnecessary activity. Furthermore, justify and appraise the benefits of acquiring and developing mathematical skills will help to decrease the bias towards proofs. Indeed, for any future professional, analytical and problem solving skills, as different points of view to face a problem, are indispensable skills that can be build and reinforced in mathematical courses.

2.2 The Meaning of Proofs in Mathematical Education [Reid(2005)]

This study focus on the inner concept of proof, the purpose of teaching mathematical proofs and the kind of reasoning that proving involves. The concept of a proof depends on the context. A formal logical deductive proof should be different to teaching a proof; in fact, the first one is more rigorous and it inherits a status of *a priori universal validity*. On the other hand, teaching proofs should be a quasi-empirical method: Instead of showing the students *what* the proof is, it should be an analytical process that goes from the exemplification to the abstraction, and students should learn *how* is it proved and *why* the proof is made in that particular way.

Regarding the purpose of teaching mathematical proofs, this study suggests that these processes will develop on students the ability to think critically in several domains of explanation. They also have a critical thinking purpose, suggesting that showing students the limits of mathematics would make them more critical of the use of numerical arguments in other domains. Finally, teaching proofs should not be strictly deductive as the evolved reasoning of the instructor has became, but involving empirical and a combination of other kind of reasoning will boost student acceptance and understanding of proofs. The goal is to provide a structure of proving as follows: verification, exploration, explanation, systematization, communication and social acceptance.

2.3 Conjecturing, generalizing and justifying: Building theory around teacher knowledge of proving [Lesseig(2016)]

This study is based on some of the practices developed in the process of proving, these practices consists of *conjecturing, generalizing and justifying*. The step of conjecturing is the ability to develop statements that could be true, generalizing is to extend hypothesis to valid statements or claims using mathematical language and abstractions, and finally justifying is the act of developing arguments to demonstrate the truth (or falsehood) of a claim using mathematical forms of reasoning. To teach students to develop knowledge and practice on these crucial steps to successfully proving, the study found that the lack of knowledge required of teachers to adequately support students understanding and engagement in these practices. Recognizing the importance of promoting these disciplinary practices in classroom communities allows that students at all levels can engage, motivate and obtain opportunities in a supportive environment.

There is a research trend called mathematical knowledge needed for teaching (MKT) about how mathematical knowledge is required and used to meet the demands of classroom teaching and to identify mathematical and pedagogical content knowledge central to the work of teaching proofs.

In this study, it is defined a two-subject-matter for teaching proofs; namely, the *Common Content Knowledge (CCK)* as the ability to solve mathematical problems; in other words, to construct proofs for oneself, and the *Specialized Content Knowledge (SCK)* as the knowledge necessary to support teaching mathematics to new learners, and it is a useful way to conceptualize the range of job-specific knowledge required.

3 Personal Reflection

The teaching and learning challenge that I decided to focus on this study is the one of teaching mathematical proofs to undergraduate students. As the three main references suggest, learning to do mathematical proofs for students is important not only for the success during the courses and get higher grades, but also for them as future professional in any area. This knowledge generates appropriate tools to face, model and solve problems; as well as it enhances critical and deductive thinking to support and defend conjectures or hypothesis. Finally, for those that purse a research career, providing valid proofs is an essential skill that prospective students must develop.

This is an unique challenge for the discipline given that courses in mathematics provide both the abstract and applicable setting to broad theories. As mathematics is one (or the most) abstract science, to plant an analytical and general deductive thinking on students should be this the place to start. Furthermore, to fight against the low-rate of skilled professionals in STEM careers, it is necessary to both teach and motivate students so they engage and enjoy doing proofs in their learning experience.

In the annotated bibliography, investigators have developed studies for the learning of proofs from the student perspective as well as the teaching and the difficulties experienced in this process by both students and instructors. However, the more general difficulties are presented to the students are due to a lack of exposition to proofs before university education and a also to a focus in multiple choice questions and short answers in first-year courses. This is translated into the fact that many students are not able to develop abstract thinking and changing the bias towards the proofs even before accessing to them. Given that a primary instructional goal is to support students progression toward deductive proof, conceptualizing proving activity in this way has great educative value. A major aim in mathematics instruction is to move students from inductive justifications towards deductive proof. Teachers productively use examples to make, test, and refine conjectures and then began to develop mathematical arguments to justify their conclusions. Teacher's knowledge is a first step to support students in the fundamental mathematical practices of conjecturing, generalizing and justifying; however, there should be a clear distinction for instructors between formal proof, those presented in a research environment, and teaching proofs, the approach that should be taken in the classroom.

Current and future mathematics instructors should take in account this challenge and realize the severity of the same, considering that traditional teaching and assessing methods might not be the most appropriate way for students to acquire deductive and reasoning skills. Also, there should be an awareness of this problem in a broader level; namely, not only students in STEM programs and instructors from mathematics courses, but also interdisciplinary discussion will help to attack these challenges. For example, in most universities there is no course that teach about proofs for a general audience; more precisely, such a course is intended for students pursing a Mathematics mayor, and for the remaining students this knowledge is barely offered on different courses. For these reasons, the support of the University and instructors is essential to develop appropriate tools to address all these challenges.

For future teaching, it is important to leave the traditional teaching and assessment methods. Implementing active learning activities that engage students into proofs by leading them to conjecture, analyse and proof important mathematical facts that appears in any level in mathematics. Mathematics is a discipline that is getting late into transitioning and implementing modern teaching techniques using any of the several available technological tools that helps students to learn and the instructor to *make mathematics not boring*; however, starting for one-self and the colleagues around us is the first step towards a huge change. Students tend to think that mathematics is a difficult subject, have negative feelings towards the subject and it is only suitable for genius; in fact, one does not need to be a genius to enjoy and do mathematics, and there is no such a thing as *"I am not a math person"*[1].

References

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[1] YouTube video by Rachel Thomas, Instructor at the University of San Francisco `https://www.youtube.com/watch?v=q6DGVGJ1WP4`