Math 142: Calculus II

Final

December 12, 2021

NAME (please print legibly): _____

Your University ID Number: _____

Instructions:

1. Read the notes below:

   - **The presence of any electronic or calculating device at this exam is strictly forbidden, including (but not limited to) calculators, cell phones, tablets and iPods.**

   - Notes of any kind are strictly forbidden.

   - **Put your FINAL answers in the boxes provided.**

   - **You must show your work to get full credit, but do not put it in the final answer box.**

   - You are responsible for checking that this exam has all 20 pages. **Do not** tear off pages from the exam.

2. Read the following Academic Honesty Statement and sign:

   I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

   Signature:_____

Part A		
QUESTION	VALUE	SCORE
1	8	
2	8	
3	8	
4	8	
5	8	
6	6	
7	8	
8	6	
9	4	
TOTAL	64	

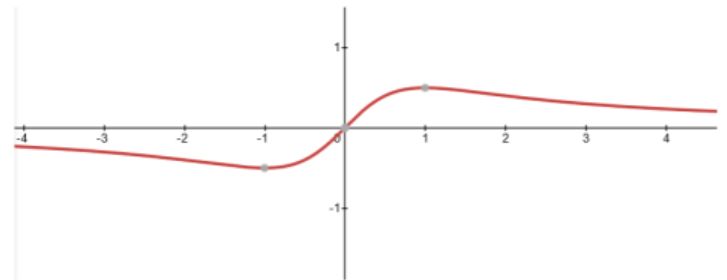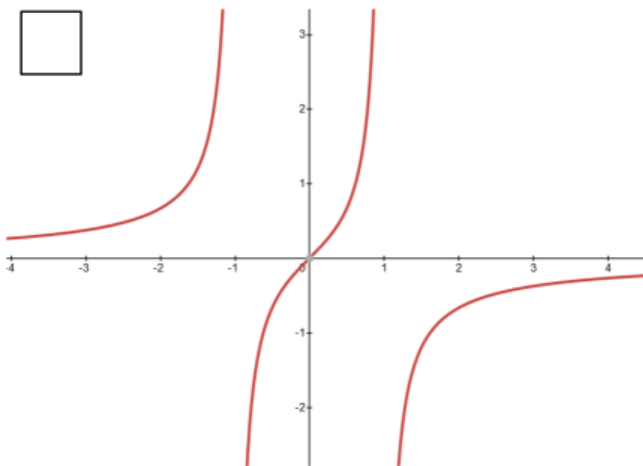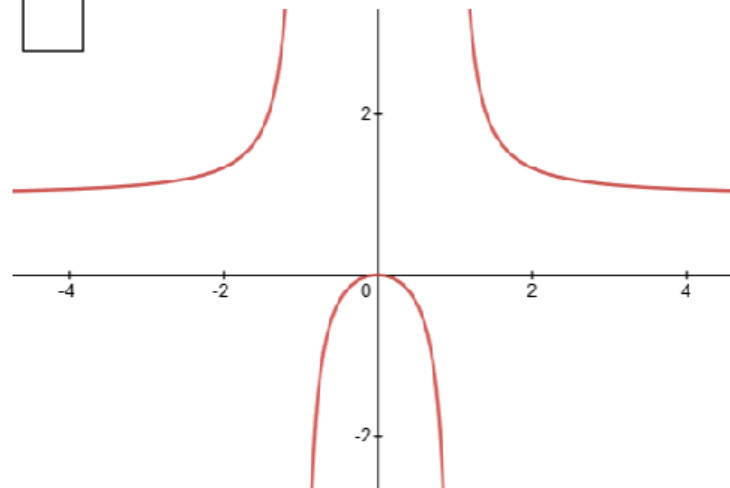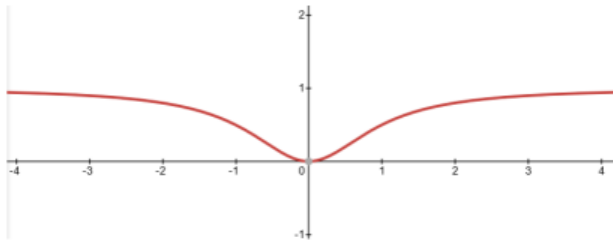| Part B		
QUESTION	VALUE	SCORE
1	8	
2	8	
3	8	
4	12	
TOTAL	36	

Part A

1. (8 points)

Match each function with its graph by writting the letter in the corresponding box. No justification is needed.

$$A. \quad \frac{x}{x^2 + 1} \qquad\qquad B. \quad \frac{x}{x^2 - 1}$$

$$C. \quad \frac{x^2}{x^2 + 1} \qquad\qquad D. \quad \frac{x^2}{x^2 - 1}$$

2. (8 points)

A manufacturer wants to construct a box without top whose base length is three times the base width. The material used to build the bottom cost $\$10/ft^2$ and the material to build the sides cost $\$5/ft^2$. If the box must have volume $60ft^3$, find the dimensions of the box that minimize the manufacturer cost.

Answer:

3. (8 points) A 30 kilogram cable is 10 meters long and hangs vertically from the top of a tall building. In this problem you may take the acceleration of gravity to be $10m/s^2$.

(a) How much work is required to lift the cable to the top of the building?

Answer:

(b) How much work is required to pull only 3 meters of the cable to the top?
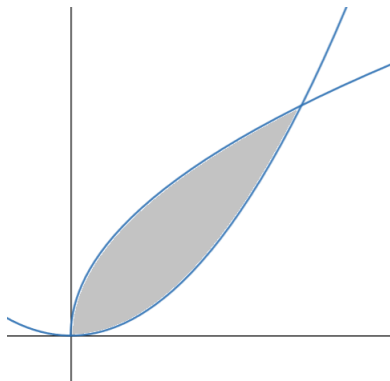
Answer:

4. (8 points)

(a) Evaluate $\displaystyle\int_0^1 \frac{e^x}{\sqrt{1+e^x}}\, dx.$

Answer:

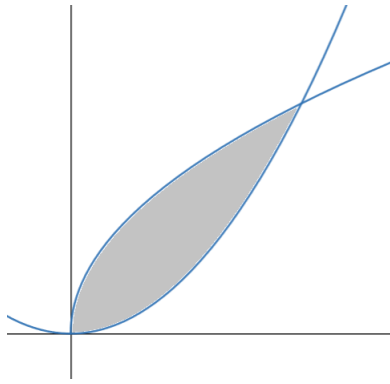(b) Evaluate $\int x \sec^2(x) \ dx$.

Answer:

5. (8 points) Consider the region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$. A sketch of the region is shown in the following picture and also on the next page.

(a) Set up (do not evaluate) an integral that represents the area of the region.

Answer:

(b)

Set up (do not evaluate) an integral that represents the volume of the solid obtained by revolving the area about the x-axis.

Answer:

6. (6 points) Use a trigonometric substitution to evaluate the integral $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\, dx$.

Answer:

7. (8 points)

(a) Evaluate the integral $\int \ln x \; dx$.

Answer:

(b) Let $F(x) = \int_x^{3x} e^{t^2} \; dt$. Find $F'(x)$.

Answer:

8. **(6 points)** The velocity of a moving particle (in ft/s) at t seconds is given by the function $v(t) = \dfrac{1}{\sqrt{t+1}}$ for $0 \le t \le 8$.

(a) Find the distance traveled by the particle.

Answer:

(b) Find the average velocity of the particle.

Answer:

9. (4 points) Evaluate the following integral using geometry only.

$$\int_{-2}^{2} 5 + \sqrt{4 - x^2} \; dx$$

Answer:

Part B

1. (8 points) Set up (do not evaluate) an integral that represents the arc length of each of the following curves.

(a) $y = \tan(x)$ for $0 \le x \le \pi/4$.

Answer:

(b) $x = ye^y$ for $0 \le y \le 1$.

Answer:

2. (8 points) Determine if the following improper integrals are convergent or divergent. Write "convergent" or "divergent" in the box. You must show all of your work.

(a) $\displaystyle\int_1^5 \frac{1}{\sqrt{x-1}}dx.$

Answer:

(b) $\displaystyle\int_1^e \frac{1}{x\ln(x)}dx.$

Answer:

3. (8 points) Evaluate each of the following improper integrals. (They both converge.)

(a) $\int_0^\infty xe^{-x}dx$.

Answer:

(b) $\displaystyle\int_0^\infty \frac{1}{x^2+1}dx.$

Answer:

4. (12 points) Evaluate each of the following integrals

(a) $\displaystyle\int \frac{x}{x^2 - 4}\,dx$

Answer:

(b) $\displaystyle\int \frac{x}{(x-1)^2}\,dx$

Answer:

(c) $\displaystyle\int \frac{6e^x}{e^{2x} + 3e^x}\,dx$

Answer: