

Math 150
Written HW 9
Due Dec 8, 2022 - 11:59 PM

Submit your solutions on Gradescope. You must show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- (1) A standard NY vehicle plate contains 3 letters followed by a 4 digits. Count the number of plates that
 - a. Do not have repeated letters or numbers.
 - b. Its letters are in alphabetical order (repetitions allowed).
 - c. The sum of the digits on the plate is divisible by 5.
- (2) Solve the following counting problems:
 - (a) A spider has one sock and one shoe on each of its eight legs, In how many different orders can the spider put on its socks and shoes? (Assume that a shoe must be on top of a sock).
 - (b) An elevator starts at the basement with 10 people and discharges them all by the time it reaches the top floor, number 6. In how many ways could have the people get off the elevator if it only matters the number of people that left on each floor?
- (3) Consider a group of n people. Determine, **in two ways**, the number of possible selections of a committee (of any size ≥ 1) and a chairperson for the committee to show that

$$\sum_{k=1}^n \binom{n}{k} k = n2^{n-1}$$

Explain the steps taken to obtain each of the elements on this identity.

- (4)
 - a. Compute $(1.2)^5$ using the expansion of $(1+x)^5$. **You won;t receive credit if using a calculator or other computational tools.**
 - b. Find the constant term of the expansion of

$$\left(\frac{x}{3} + \frac{2}{\sqrt{x}}\right)^{15}$$

- (5) Each year, 40% of a salmon population is extracted from a farming pond. At the beginning of the next year, the pond is restocked with an additional fixed amount of N caught wild salmon. Let P_n denote the amount of fish at the beginning of the n -th year Assume that the initial salmon population on the pond is $P_0 = 5000$
 - a. Write a recursion to describe P_n .
 - b. Determine the value of N so the amount of fish remains constant at the beginning of each year.